

Scattering, degeneracy and Pauli operators

GRE · GRE Subject Test · GRE Physics

Scattering, degeneracy and Pauli operators

A quantum wave can reflect from a downward potential step even though a classical particle has no turning point there.

Prerequisites: 5.

- Match travelling waves and fluxes across finite potential changes
- Count degenerate isotropic-oscillator states
- Evaluate Pauli products and distinguish anticommutation from equality

Choose the system and model

For constant potential V and energy $E > V$, the spatial solutions are travelling factors $\exp(\pm ikx)$, with $k = \sqrt{2m(E - V)}/\hbar$. At finite steps with the same particle mass, wavefunction and its first derivative are continuous. In a left-incident scattering problem with no incoming beam from the right, the far-right solution contains only the right-travelling factor Ae^{ikx} . If a finite well returns to the original external potential, the transmitted external wave number equals the incident one, even though the interior wave number differs. Exponentially decaying solutions describe $E < V$ regions, not every potential well.

Use the governing relation

For a step from V_1 to V_2 with both regions classically allowed, write incident-plus-reflected amplitude in region 1 and transmitted amplitude in region 2. Continuity gives $r = (k_1 - k_2)/(k_1 + k_2)$ and $t = 2k_1/(k_1 + k_2)$. Reflection probability is $R = |r|^2$; transmission is $T = (k_2/k_1)|t|^2$ because probability current depends on wave number. Thus $R + T = 1$ for a lossless step. Do not add raw squared transmitted amplitude to R without its current factor. With $k_2 = 3k_1$, $R = 1/4$ and $T = 3/4$ despite a downward step.

Apply the conditions

A three-dimensional isotropic oscillator separates into x, y, z modes with nonnegative integers n_x, n_y, n_z . Its energy is $(N + 3/2)\hbar\omega$, where $N = n_x + n_y + n_z$. For a spin-zero distinguishable single particle, the spatial degeneracy is the number of such triples: $(N + 1)(N + 2)/2$. This counts different assignments, not just different unordered partitions. For $N = 2$, the six states are permutations of $(2, 0, 0)$ and $(1, 1, 0)$. For $N = 3$, the degeneracy is 10. Additional spin or identical-particle constraints would change the counting problem.

Check the conclusion

Pauli matrices are $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Each squares to identity. Direct multiplication gives $\sigma_x \sigma_y = i \sigma_z$ and $\sigma_y \sigma_x = -i \sigma_z$, so distinct Pauli matrices anticommute. Cyclic products $x \rightarrow y \rightarrow z$ have positive i ; reversing order changes the sign. The general identity is $\sigma_i \sigma_j = \delta_{ij} I + i \sum_k \epsilon_{ijk} \sigma_k$. Matrix order matters: $\sigma_x \sigma_z = -i \sigma_y$, not $i \sigma_y$ or simply σ_y . These dimensionless matrices become spin operators $S_i = \hbar \sigma_i / 2$, adding physical units and factors.

Worked method

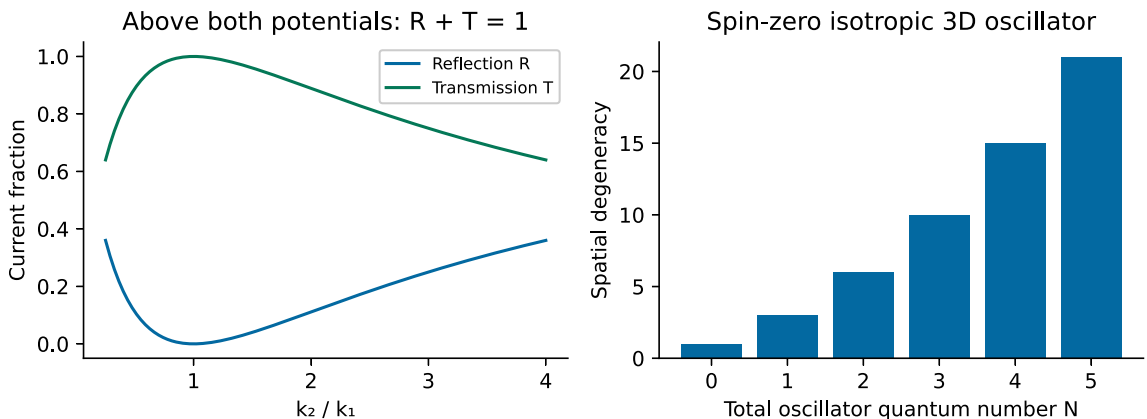
At a finite potential step, match wavefunction and derivative for the same particle mass.

$$r = \frac{k_1 - k_2}{k_1 + k_2}, \quad t = \frac{2k_1}{k_1 + k_2}.$$

Transmission probability uses current, not amplitude alone. For $k_2 = 3k_1$, $r = -1/2$, $t = 1/2$.

$$R = |r|^2 = 1/4, \quad T = (k_2/k_1)|t|^2 = 3/4, \quad R + T = 1.$$

Reflection occurs even for a downward step. Its phase sign disappears when taking probability.



Scattering, degeneracy and Pauli operators: original GRE teaching diagram.

Check conditions and vocabulary

Transmission probability needs the wave-number current ratio. Oscillator degeneracy counts ordered mode triples. Do not reverse Pauli matrix order without changing its sign.

reflection coefficient : Reflected probability-current fraction relative to incident current.

degeneracy: Number of independent states sharing the specified energy.