

Relativistic lifetime, energy and Doppler shift

GRE · GRE Subject Test · GRE Physics

A fast unstable particle can travel farther in the laboratory than its rest-frame lifetime multiplied by its speed would suggest.

Prerequisites: 6, 42.

- Relate proper lifetime to laboratory time and distance
- Use invariant energy–momentum and relativistic kinetic energy
- Infer longitudinal recession speed from wavelength ratio

Choose the system and model

For relative speed v , define $\beta=v/c$ and $\gamma=1/\sqrt{1-\beta^2}$. Proper time is measured along the particle's worldline by a clock at rest with it. If its proper mean lifetime is τ_0 , its laboratory mean lifetime is $\gamma\tau_0$ and mean travel distance is $v\gamma\tau_0$ at constant speed. The decay is statistical: mean lifetime is not a guaranteed decay time for each particle. At $\beta=0.8$, $\gamma=5/3$. Compute spacetime events consistently in one frame; proper time and coordinate time are different quantities.

Use the governing relation

Total energy is $E=\gamma mc^2$, momentum $p=\gamma mv$ and invariant $E^2-p^2c^2=m^2c^4$. When E and pc are expressed in the same energy units, find $mc^2=\sqrt{E^2-(pc)^2}$, not $E-pc$. The nonnegative square root is required for positive rest mass. For $E=13$ GeV and $pc=12$ GeV, rest energy is 5 GeV and mass is 5 GeV/ c^2 . A massless particle has $E=pc$ but need not have zero energy or momentum. These relations concern isolated-particle four-momentum, not classical mv at relativistic speed.

Apply the conditions

Work accelerating a particle from rest equals kinetic energy $K=E-mc^2=(\gamma-1)mc^2$. At $\beta=0.6$, $\gamma=1.25$ and $K=0.25mc^2$, while the classical $\frac{1}{2}mv^2$ gives $0.18mc^2$. Classical kinetic energy is the low-speed expansion and becomes inaccurate near c . Finite acceleration work increases γ rather than allowing a massive particle to reach or exceed c . Keep total energy, rest energy and kinetic energy distinct when interpreting answer units.

Check the conclusion

For purely longitudinal relative recession in special relativity, wavelength ratio $r = \lambda_{\text{observed}} / \lambda_{\text{emitted}} = \sqrt{(1+\beta)/(1-\beta)}$. Rearranging gives $\beta = (r^2 - 1)/(r^2 + 1)$. A ratio $r=2$ gives $\beta=3/5$, not c times $r-1$ from a low-speed approximation. Blueshift uses $r < 1$ and a negative recession parameter under this convention. The formula assumes the shift is entirely kinematic; cosmological expansion or gravitational redshift requires a different model. State the question's stipulated model before interpreting a spectral ratio.

Worked method

Total energy and momentum determine invariant rest energy.

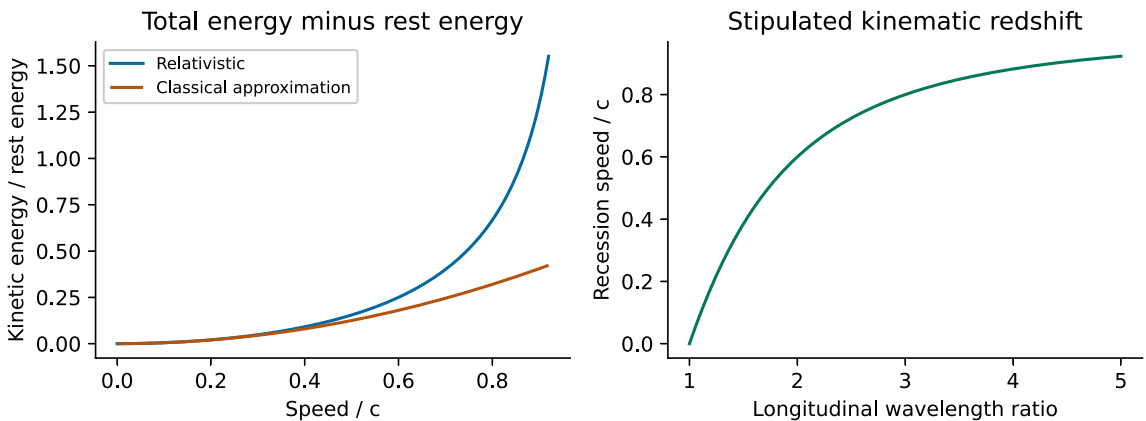
$$E^2 - (pc)^2 = (mc^2)^2, \quad mc^2 = \sqrt{E^2 - (pc)^2}.$$

For $E = 17 \text{ GeV}$ and $pc = 15 \text{ GeV}$,

$$mc^2 = \sqrt{(17 \text{ GeV})^2 - (15 \text{ GeV})^2} = 8 \text{ GeV}.$$

$$K = E - mc^2 = 17 \text{ GeV} - 8 \text{ GeV} = 9 \text{ GeV}.$$

Rest mass is $8 \text{ GeV}/c$ squared. Do not subtract momentum directly from energy.



Relativistic lifetime, energy and Doppler shift: original GRE teaching diagram.

Check conditions and vocabulary

Do not use the proper lifetime as laboratory time, subtract pc from E to find rest mass, or apply the classical Doppler approximation to a large wavelength ratio.

proper time: Time recorded by a clock moving with the object along its worldline.

rest energy: Energy mc^2 associated with an object's rest mass.