

Gas processes, entropy and reversible cycles

GRE · GRE Subject Test · GRE Physics

A refrigerator can move more heat than the work supplied to it. That ratio is a coefficient of performance, not an efficiency that must be below one.

Prerequisites: 4.

- Apply first-law work and heat signs to gas processes
- Compare reversible entropy balances and P–V cycle areas
- Distinguish engine efficiency from refrigerator coefficient of performance

Choose the system and model

Use $\Delta U = Q - W$, where Q is heat entering the system and W is work done by it. Quasistatic boundary work is $\int P \, dV$, positive during expansion. For a fixed amount of ideal gas, internal energy depends only on temperature. At constant volume $W=0$, so $\Delta U=Q$; for an isothermal ideal-gas process $\Delta U=0$, so $Q=W$. Adiabatic means $Q=0$, not constant temperature. Over a cycle, the state returns and $\Delta U=0$, so net Q equals net W even though individual legs have different heat transfers.

Use the governing relation

For reversible isothermal expansion of an ideal gas, PV is constant. With constant heat capacities, a reversible adiabatic path obeys $PV^\gamma = \text{constant}$, with $\gamma = C_p/C_v > 1$; through the same initial state its pressure falls faster as volume increases. An isobaric path is horizontal on a P–V plot. A clockwise closed cycle has positive $\oint P \, dV$; reversing direction changes the sign. For a rectangular loop the magnitude is $\Delta P \Delta V$, and a triangular loop has half the corresponding bounding-rectangle area. Use Pa and m³ for joules, not an unconverted litre value.

Apply the conditions

For reversible heat transfer, $dS = \delta Q_{\text{rev}}/T$; entropy is a state function while heat is path dependent. A reversible system-plus-environment process has zero total entropy change, but the system's entropy alone can increase or decrease. Irreversible spontaneous processes produce nonnegative total entropy. A reversible isothermal expansion increases the gas entropy by $nR \ln(V_2/V_1)$; the reservoir loses the same amount. Reversibility does not require constant system temperature, zero work or zero internal-energy change.

Check the conclusion

A reversible engine between T_H and T_C has efficiency $W/Q_H = 1 - T_C/T_H$. A reversible refrigerator instead has $\text{COP} = Q_C/W = T_C/(T_H - T_C)$, and a heat pump has $Q_H/W = T_H/(T_H - T_C)$. Use absolute kelvin temperatures. On a reversible T - S diagram, heat magnitude along an isotherm is T times the entropy change; the Carnot rectangle's area is the work magnitude. Refrigeration moves heat from cold to hot by consuming work, reversing the engine cycle. COP can exceed one without violating conservation because the moved heat is not supplied solely by the work.

Worked method

A reversible refrigerator transfers entropy ΔS from T_c to T_h .

$$Q_c = T_c \Delta S = (270 \text{ K})(3.0 \text{ J/K}) = 810 \text{ J}.$$

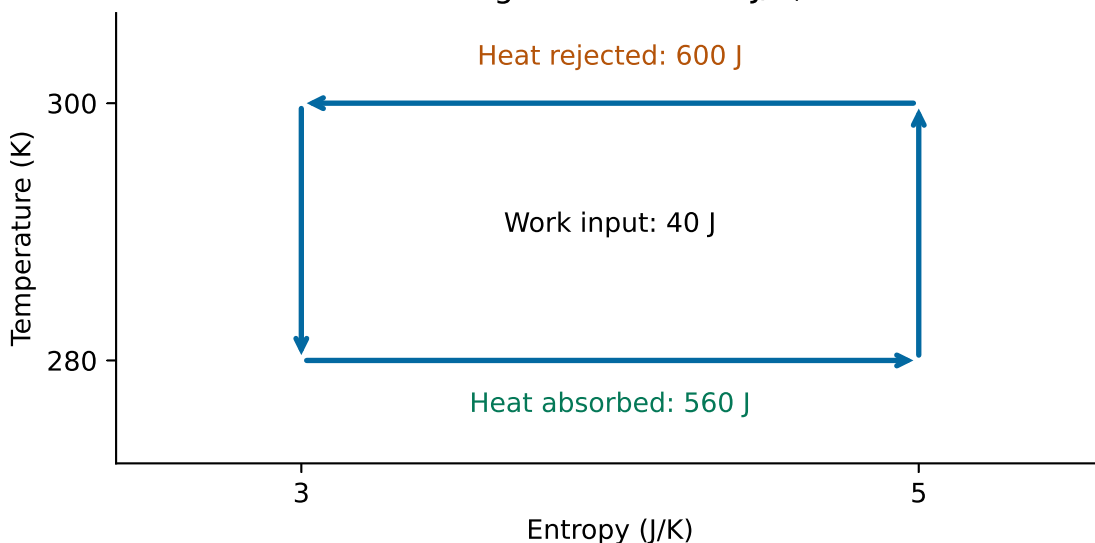
$$Q_h = T_h \Delta S = (300 \text{ K})(3.0 \text{ J/K}) = 900 \text{ J}.$$

$$W_{in} = Q_h - Q_c = 900 \text{ J} - 810 \text{ J} = 90 \text{ J}.$$

$$\text{COP} = Q_c/W_{in} = (810 \text{ J})/(90 \text{ J}) = 9.$$

COP above one is possible because heat is moved, rather than all produced from work.

Reversible refrigerator: $\Delta S = 2 \text{ J/K}$; $\text{COP} = 14$



Gas processes, entropy and reversible cycles: original GRE teaching diagram.

Check conditions and vocabulary

Zero total entropy production does not mean zero system entropy change. A refrigerator uses $T_C/(T_H - T_C)$, not the engine efficiency formula.

entropy production: Nonnegative total entropy generated by irreversibility.

coefficient of performance: Useful heat transferred divided by work input for a refrigerator or heat pump.