

Fourier symmetry and wave superposition

GRE · GRE Subject Test · GRE Physics

A waveform can contain many harmonics while having no sine terms at all. Reflection symmetry tells you which coefficients vanish before integration.

Prerequisites: 3, 47.

- Use orthogonality to identify Fourier coefficients
- Exploit even and odd parity without discarding the constant term
- Relate component amplitudes to interference and physical waveforms

Choose the system and model

For a sufficiently regular real 2π -periodic function, write $f(x) = a_0/2 + \sum [a_n \cos(nx) + b_n \sin(nx)]$. The coefficients are $a_n = (1/\pi) \int_{-\pi}^{\pi} f(x) \cos(nx) dx$ and $b_n = (1/\pi) \int_{-\pi}^{\pi} f(x) \sin(nx) dx$, with a_0 using $n=0$. Cosines and sines are orthogonal on a full period. The mean is $a_0/2$, not a_0 ; retain the constant term even when all nonzero-frequency coefficients of one family vanish.

Use the governing relation

If f is even about the chosen origin, $f(x)\sin(nx)$ is odd and its symmetric integral is zero, so all b_n vanish. If f is odd, its mean and all a_n vanish. Symmetry depends on the origin: shifting the same physical signal can mix sine and cosine coefficients while leaving its harmonic frequencies unchanged. A nonnegative triangular waveform symmetric about $x=0$ can therefore have cosine harmonics and a nonzero mean without any sine harmonics.

Apply the conditions

For the 2π -periodic extension of $f(x)=|x|$ on $[-\pi, \pi]$, the mean is $\pi/2$. Integrating $x \cos(nx)$ by parts on $[0, \pi]$ gives $a_n = 2[(-1)^n - 1]/(\pi n^2)$, so even- n cosine coefficients are zero and odd- n coefficients are $-4/(\pi n^2)$. All sine coefficients are zero. The $1/n^2$ decay reflects a continuous function with a slope discontinuity. A jump discontinuity often gives slower coefficient decay and partial-sum overshoot; do not infer the same convergence behaviour for every waveform.

Check the conclusion

A Fourier sum adds amplitudes with their phases. For equal coherent monochromatic amplitudes A meeting with phase difference φ , resultant squared amplitude is $2A^2(1+\cos\varphi)$, so intensity is $2I_0(1+\cos\varphi)$. It is $4I_0$ in phase and zero at $\varphi=\pi$. Incoherent averaging removes the cross term, yielding $2I_0$. Harmonics at different frequencies can construct a shape over time; their instantaneous sum is not the sum of their in-

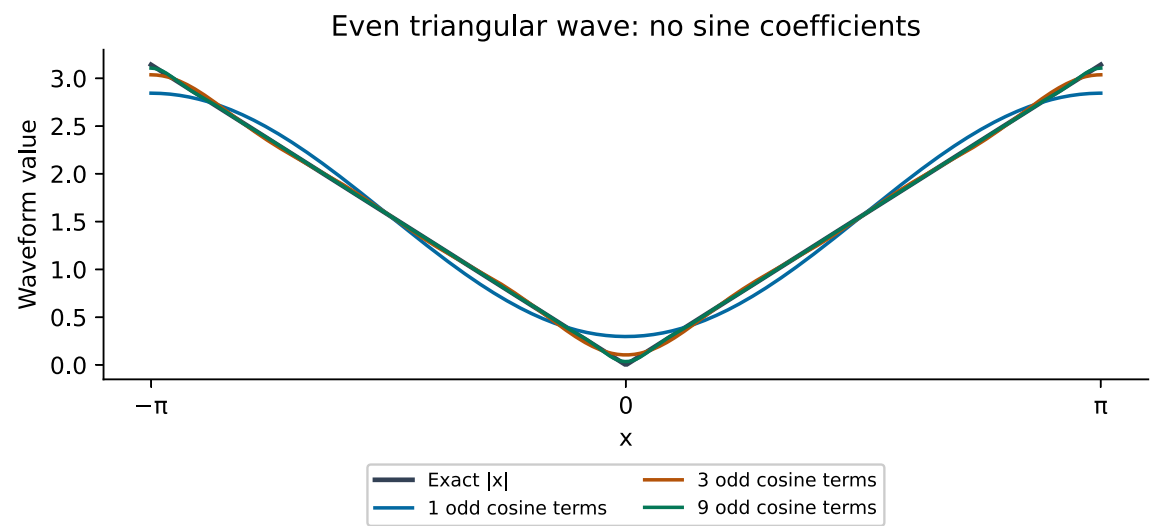
dividual intensities. Identify whether the question concerns a waveform, time-average power or coherent interference.

Worked method

For a real periodic function, use $f = a_0/2 + \sum (a_n \cos nx + b_n \sin nx)$. Even symmetry makes all sine coefficients zero because the integrand is odd. For $f(x) = |x|$ on $[-\pi, \pi]$,

$$a_n = \frac{2}{\pi} \int_0^\pi x \cos(nx) \, dx = \frac{2[(-1)^n - 1]}{\pi n^2}.$$

Integration by parts supplies the second expression. The mean is $a_0/2 = \pi/2$; only odd cosine harmonics survive. A cusp does not justify dropping the constant term.



Fourier symmetry and wave superposition: original GRE teaching diagram.

Check conditions and vocabulary

Even symmetry removes sine coefficients, not the constant term or every cosine harmonic. Check the symmetry origin and whether fields are coherent before adding intensities.

orthogonality: A zero integral of the product of distinct basis functions over the specified interval.

Fourier coefficient : Weight of a sine, cosine or constant basis component in a periodic expansion.