

Acoustic Doppler echoes and standing-wave boundaries

GRE · GRE Subject Test · GRE Physics

A driver hears an echo from a stationary wall at a shifted pitch: the outbound and returning sound each acquire a Doppler factor .

Prerequisites: 3.

- Apply source and observer Doppler factors in the medium frame
- Derive allowed pipe frequencies from displacement boundary conditions
- Distinguish frequency, wavelength and boundary changes

Choose the system and model

For sound in a stationary medium, separate source motion from observer motion. A source approaching at speed u_s compresses wavefront spacing and produces received frequency $f c/(c-u_s)$ at a stationary observer. An observer approaching a stationary source at speed u_o meets more wavefronts per second and measures $f(c+u_o)/c$. Speeds are measured relative to the medium; the acoustic source and observer formulas are not symmetric under exchanging roles. Receding motion reverses the appropriate sign. These expressions assume subsonic motion along the propagation line.

Use the governing relation

For a siren moving toward a stationary reflecting wall with speed u , the wall first receives $f_{\text{wall}} = f c/(c-u)$. Reflection from a stationary wall preserves frequency in the medium frame. The moving driver then approaches the returning wavefronts and receives $f_{\text{echo}} = f_{\text{wall}}(c+u)/c = f(c+u)/(c-u)$. At small u/c the fractional shift is approximately $2u/c$, but the exact expression has different numerator and denominator. A moving reflecting surface needs its own Doppler step; do not reuse the stationary-wall result blindly.

Apply the conditions

At an ideal open pipe end, air displacement is an antinode and pressure variation is a node. At a rigid closed end, displacement is a node and pressure is an antinode. For both ends open, length L contains n half-wavelengths, giving $f_n = nc/(2L)$, $n=1,2,\dots$. For one end closed, it contains an odd number of quarter-wavelengths, giving $f_n = (2n-1)c/(4L)$. Real pipes may need end corrections; the ideal GRE model uses the stated length without inventing an adjustment.

Check the conclusion

Closing one end of a previously both-open pipe halves its fundamental and leaves only odd multiples of that new fundamental. Its old frequencies were integer multiples of $c/(2L)$, which are even multiples of $c/(4L)$, so none is an allowed frequency of the new ideal one-closed spectrum. This differs from simply deleting even harmonics while retaining the old fundamental. When the medium is unchanged, wave speed is unchanged; frequency and wavelength change together to satisfy the new boundary geometry.

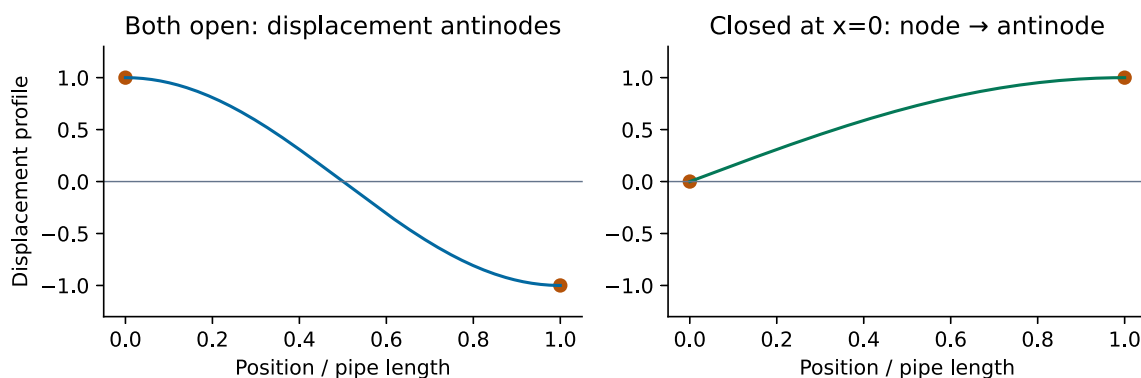
Worked method

A siren approaches a stationary wall in still air. Treat outward and return journeys separately. The wall first receives $f_w = f c / (c - u)$; reflection keeps this frequency in the medium frame. The moving listener then receives $f_e = f_w (c + u) / c$.

$$f_e = f \frac{c + u}{c - u} = (600 \text{ Hz}) \frac{340 \text{ m/s} + 20 \text{ m/s}}{340 \text{ m/s} - 20 \text{ m/s}} = 675 \text{ Hz}.$$

This Doppler effect uses velocities relative to the medium. It is not the light-wave formula.

Ideal fundamental modes; $f_{\text{closed}} = f_{\text{open}} / 2$



Acoustic Doppler echoes and standing-wave boundaries: original GRE teaching diagram.

Check conditions and vocabulary

An echo needs both Doppler factors. Closing a pipe changes the fundamental; odd harmonics are counted from the new fundamental, not the old one.

displacement antinode : A standing-wave position where the air displacement amplitude is maximal.

Doppler factor: Frequency multiplier produced by a specified source or observer motion.