

# 37.1 Weak perturbations and degenerate subspaces

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 11 marks****KEY WORDS** first-order energy shift

- degenerate perturbation theory

## Objective

Calculate nondegenerate first-order energy shifts as normalised expectation values; Use symmetry and matrix elements to assess weak-coupling limits; Diagonalise the perturbation within an exactly degenerate subspace . Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

For  $H = H_0 + \lambda W$  and normalised nondegenerate  $|n\rangle$ , first-order shift is  $\lambda \langle n|W|n\rangle$ . A weak  $\gamma x$  potential in  $0 < x < L$  gives  $\gamma L/2$ , not the potential at a chosen point.

Centred odd perturbations have zero diagonal expectation for a parity state. A centred quadratic shift is  $\eta L^2[1/12 - 1/(2\pi^2 n^2)]$ . Zero first order does not imply an unchanged exact state.

Weak mixing needs  $|\lambda W_{mn}|$  small compared with  $|E_n^0 - E_m^0|$ . Second-order terms use squared couplings divided by signed gaps; the ground-state denominators are negative.

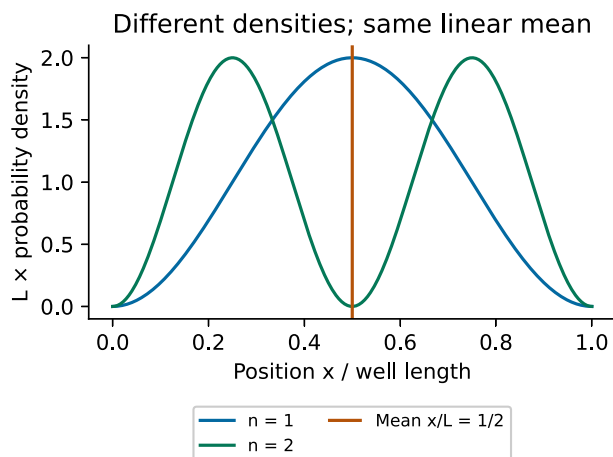
For exact degeneracy, diagonalise the perturbation matrix within the subspace.  $\epsilon[[2,1],[1,2]]$  has shifts  $\epsilon$  and  $3\epsilon$ , from antisymmetric/symmetric combinations. Diagonal entries alone are insufficient.

### ■ Worked method

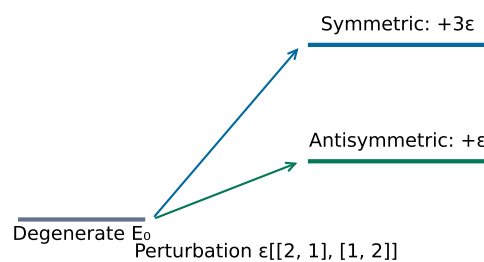
A degenerate subspace requires matrix diagonalisation before first-order shifts. Suppose the perturbation restricted to two states is  $\epsilon \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ .

$$\det(W - wI) = (2\epsilon - w)^2 - \epsilon^2 = 0.$$

Shifts are epsilon and 3 epsilon. Normalised states are  $(1, -1)/\sqrt{2}$  and  $(1, 1)/\sqrt{2}$ . Reading only the diagonal entries would miss the splitting. Couplings to other levels must remain weak relative to their energy gaps.



Diagonalise the complete degenerate block



## 2 Practice

**2.1** In  $0 < x < L$ , a weak perturbation  $\gamma(x - L/2)$  acts on an unperturbed infinite-well eigenstate. Its first-order energy shift is: [1]

- A.  $\gamma L$
- B.  $\gamma L/2$
- C. 0
- D.  $-\gamma L/2$
- E. Undefined for every  $n$

**2.2** A two-state degenerate block has perturbation  $\varepsilon[[0, 1], [1, 0]]$ ,  $\varepsilon > 0$ . First-order shifts are: [1]

- A. 0 and 0
- B.  $\varepsilon$  and  $\varepsilon$
- C.  $-\varepsilon$  and  $+\varepsilon$
- D. 0 and  $2\varepsilon$
- E.  $-2\varepsilon$  and  $2\varepsilon$

**2.3** For a 4 nm well and weak  $\gamma x$  with  $\gamma = 0.02$  eV/nm, first-order shift is \_\_\_\_\_ eV. Show your relation and units. [2]

