

36.1 Quantum occupations and limits of equipartition

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS mean occupation • equipartition

Objective

Compute mean occupation per complete quantum state for fermions and bosons; Count allowed identical-particle occupations and identify the classical dilute limit; Compare classical quadratic-mode heat capacity with a quantum oscillator response. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

One complete state: $n_F = 1/(\hat{x} + 1)$, $n_B = 1/(\hat{x} - 1)$, $x = (\epsilon - \mu)/(k_B T)$. A g -fold level has total g n . Fermionic exclusion is per complete state; photons at equilibrium have $\mu = 0$.

Two identical fermions in four complete states: $\text{choose}(4, 2) = 6$; two bosons: $\text{choose}(5, 2) = 10$; two labelled particles: 16. Spin can distinguish complete states in the same orbital.

For $\hat{x} \gg 1$, both quantum occupations approach e^{-x} . At $x = \ln 4$: fermion $1/5$, boson $1/3$, classical $1/4$. Particle count alone is not a classical-limit condition.

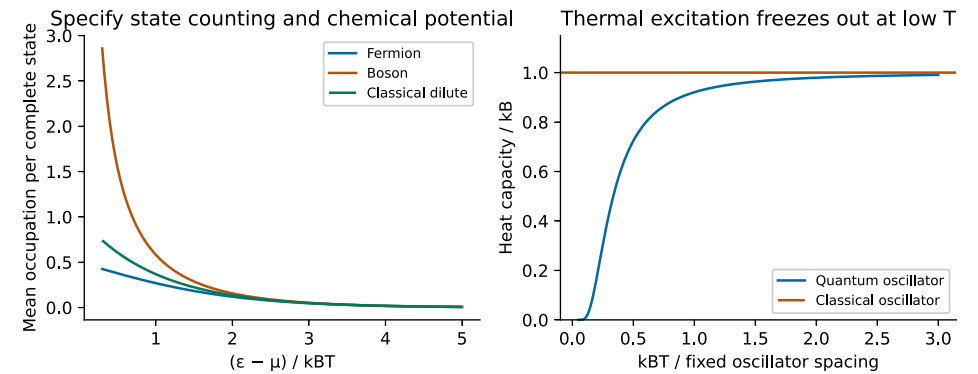
Classical quadratic terms each contribute $k_B T/2$. One 1D oscillator gives $C = k_B$; quantum $C/k_B = x^2 e^{-x} / (\hat{x} - 1)^2$ tends to one at high T and zero at low T . A fixed zero-point offset contributes no heat capacity.

■ Worked method

Occupation number refers to one complete single-particle state. For $x = (\epsilon - \mu)/(k_B T) = \ln 4$,

$$\bar{n}_F = (e^x + 1)^{-1} = 1/5, \quad \bar{n}_B = (e^x - 1)^{-1} = 1/3.$$

The classical dilute approximation gives $e^{-x} = 1/4$; it is not exact here. A g -fold level has mean total occupation $g\bar{n}$, so a fermionic level can contain more than one particle while each complete state still obeys exclusion.



2 Practice

2.1 Choose one answer. At $(\epsilon - \mu)/(k_B T) = \ln 3$, the mean occupation of one fermionic state is: [1]

- A $1/2$
- B $1/3$
- C $1/4$
- D $3/4$
- E 3

2.2 Choose one answer. Two identical fermions can occupy six distinct complete states. How many occupation patterns exist? [1]

- A 6
- B 12
- C 15
- D 21
- E 36

2.3 At $x = \ln 5$, one bosonic state has mean occupation _____ (give an exact decimal). Show your relation and units. [2]

[2]

[4]