

34.1 Thermal transport, calorimetry and expansion

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS thermal resistance • latent heat

Objective

Combine conductive thermal resistances and distinguish heat rate from heat flux; Balance sensible and latent heat with stated isolation and heat-capacity conditions; Calculate free thermal expansion and constrained thermal stress with their limits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Fourier flux is $-\kappa dT/dx$; steady slab rate $\kappa A \Delta T/L$. $R_{th}=L/(\kappa A)$ in K/W; add series R or parallel $1/R$. Rate is W; flux is W/m².

With constant specific heat, $Q=mc\Delta T$. Sum all energy changes for isolated calorimetry. A phase change needs $Q=ml$ at the transition temperature; sensible and latent energy are distinct.

Free isotropic small expansion: $\Delta L/L \approx \alpha \Delta T$, $\Delta A/A \approx 2\alpha \Delta T$, $\Delta V/V \approx 3\alpha \Delta T$. Prevented elastic expansion gives tensile-signed $\sigma \approx -Y\alpha \Delta T$ on heating. State constraints and elastic limits.

Convection moves matter; net radiation is $\varepsilon \sigma_{SB} A(T^4 - T_{env}^4)$ using kelvin. Lumped Newton cooling gives excess temperature decay $\exp(-t/\tau)$, $\tau = C/(hA)$, only when internal gradients are negligible.

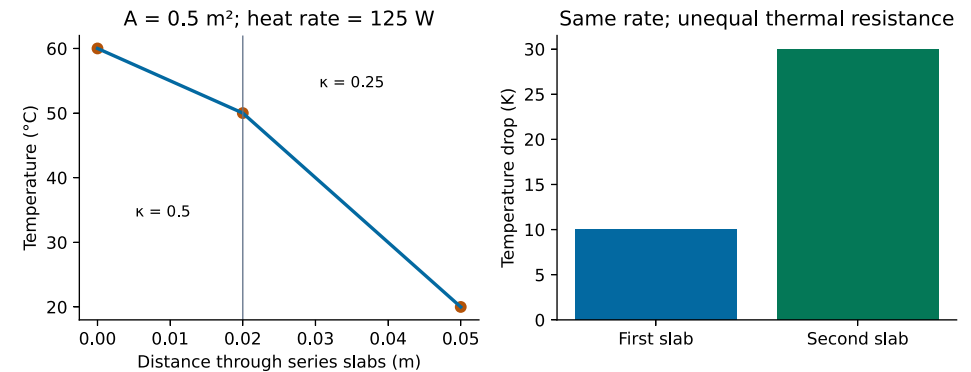
■ Worked method

Two slabs in series carry the same steady heat rate. Let resistances be 0.08 and 0.24 K/W. Boundary temperatures are 60 and 20 degrees Celsius.

$$\dot{Q} = \Delta T / (R_1 + R_2) = (40 \text{ K}) / (0.32 \text{ K/W}) = 125 \text{ W}.$$

$$\Delta T_1 = \dot{Q} R_1 = (125 \text{ W})(0.08 \text{ K/W}) = 10 \text{ K}.$$

The interface is 50 degrees Celsius when slab 1 touches the hot side. Heat rate has watt units; flux divides it by area.



2 Practice

2.1 Choose one answer. Two slabs in series without sources have thermal resistances R and 3R. The steady temperature-drop ratio is: [1]

- A 1:1
- B 3:1
- C 1:3
- D 1:9
- E Zero in both

2.2 Choose one answer. An isotropic unconstrained plate with a circular hole is warmed under small uniform expansion. Its hole radius: [1]

- A Shrinks
- B Stays fixed
- C Increases by fractional $\alpha \Delta T$
- D Increases by fractional $3\alpha \Delta T$
- E Must close

2.3 A body has $C=200 \text{ J/K}$ and $hA=10 \text{ W/K}$ under lumped Newton cooling. Its time constant is _____ s. Show your relation and units. [2]

[2]

[5]

Solutions

2.1 C Their heat rate is the same; $\Delta T_i = Q \cdot R_i$ gives 1:3.

Compare the rejected options: 1:1; 3:1; 1:9; Zero in both. Use the stated model and conditions.

2.2 C All in-plane lengths, including the hole boundary, scale by $1 + \alpha \Delta T$; volume scaling is a different quantity.

Compare the rejected options: Shrinks; Stays fixed; Increases by fractional $3\alpha \Delta T$; Must close. Use the stated model and conditions.

2.3 20. $\tau = C/(hA) = 200/10 = 20$ s.

2.4 True. Mechanical strain must cancel positive thermal strain; tensile-signed stress is $-Y\alpha\Delta T$.

3.1 Cooling water to zero releases $mc\Delta T = (0.20)(4200)(40) = 33600$ J. Melting all ice needs $m\ell = (0.050)(334000) = 16700$ J, so all ice melts. Remaining energy heats 0.25 kg water: $T_f = (33600 - 16700)/(0.25 \times 4200) = 16.10$ degrees Celsius. Award cooling budget, melt budget, phase decision, final heat capacity and temperature. If cooling energy were insufficient, final temperature would be zero with some ice left.