

# 30.1 RC and RL transients, impedance and resonance

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 11 marks****KEY WORDS** time constant • impedance

## Objective

Solve RC charging and discharging with stated initial conditions; Solve RL current response and account for stored energy; Use complex impedance, phase and resonance in sinusoidal circuits. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

For RC,  $R \, dq/dt + q/C = V_s$  and  $\tau = RC$ .  $V_C = V_s + (V_C(0) - V_s) \exp(-t/\tau)$ . An uncharged capacitor reaches 63.2% after  $\tau$ ; discharge leaves 36.8%. Voltage is continuous without impulsive current.

For RL,  $L \, dI/dt + RI = V_s$  and  $\tau = L/R$ .  $I = V_s/R + (I(0) - V_s/R) \exp(-t/\tau)$ . Current is continuous for finite voltage. Stored energy  $LI^2/2$  becomes resistor heat during decay.

Using  $\exp(i\omega t)$ , series RLC has  $Z = R + iX$  with  $X = \omega L - 1/(\omega C)$ .  $I_{\text{rms}} = V_{\text{rms}}/\sqrt{R^2 + X^2}$ ; current lags for  $X > 0$  and leads for  $X < 0$ . Add complex impedances, not magnitudes.

Series driven resonance has  $\omega_0 = 1/\sqrt{LC}$ ,  $X = 0$  and maximal current. Mean power is  $I_{\text{rms}}^2 R$ . Damped free oscillation instead has frequency  $\sqrt{1/(LC) - (R/(2L))^2}$  when underdamped. Distinguish the drive and regime.

### ■ Worked method

An RC circuit switches to fixed source voltage  $V_s$ . Charge conservation and Kirchhoff's law give

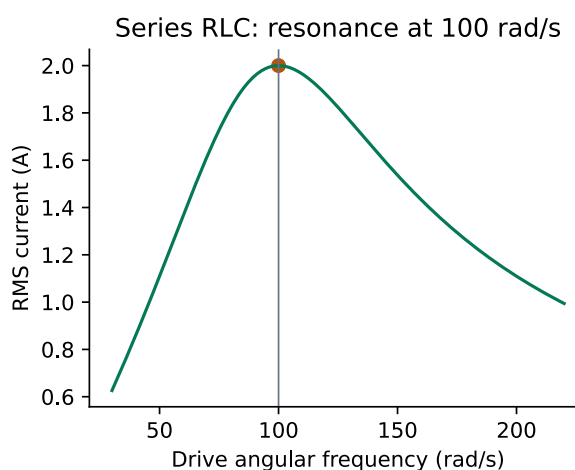
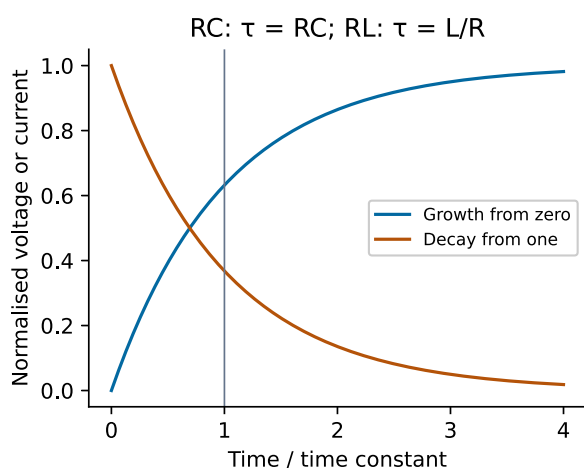
$$RC \dot{V}_C + V_C = V_s, \quad V_C(t) = V_s + [V_C(0) - V_s] e^{-t/(RC)}.$$

For  $R = 2$  kilohms,  $C = 1$  millifarad,  $V_s = 12$  V and initial voltage zero,

$$\tau = RC = (2000 \, \Omega)(0.001 \, \text{F}) = 2.0 \, \text{s}.$$

$$V_C(\tau) = V_s(1 - e^{-1}) = (12 \, \text{V})(1 - e^{-1}) = 7.59 \, \text{V}.$$

The time constant controls the remaining difference from the final state.



## 2 Practice

**2.1** Choose one answer. After one time constant, an initially uncharged RC capacitor driven by a constant source reaches approximately: [1]

- A 36.8% of final voltage
- B 50% of final voltage
- C 63.2% of final voltage
- D 86.5% of final voltage
- E 100% of final voltage

**2.2** Choose one answer. Using  $\exp(i\omega t)$ , a series circuit with  $X = \omega L - 1/(\omega C) > 0$  has: [1]

- A Current leading voltage
- B Current lagging voltage
- C Voltage and current necessarily in phase
- D Zero resistor dissipation
- E Impedance magnitude  $R + X$

**2.3** An RL circuit has  $L = 0.6$  H and  $R = 3$   $\Omega$ . Its time constant is \_\_\_\_\_ s. Show your relation and units. [2]



## Solutions

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**2.1 C**  $V_C/V_s = 1 - \exp(-1) \approx 0.632$ ;  $\exp(-1)$  is the remaining difference.

Compare the rejected options: 36.8% of final voltage; 50% of final voltage; 86.5% of final voltage; 100% of final voltage. Use the stated model and conditions.

**2.2 B**  $Z$  has a positive phase;  $I = V/Z$  has a negative phase relative to voltage.

Compare the rejected options: Current leading voltage; Voltage and current necessarily in phase; Zero resistor dissipation; Impedance magnitude  $R+X$ . Use the stated model and conditions.

**2.3 0.2.**  $\tau = L/R = 0.6/3 = 0.2$  s.

**2.4 True.** Its ideal voltage/current phase is a quarter cycle; average power is zero while  $LI^2/2$  varies.

**3.1** With  $\tau = RC$ ,  $V_C = -4 + 12e^{-t/\tau}$  volts. Zero crossing satisfies  $e^{-t/\tau} = 1/3$ , so  $t = \tau \ln 3$ .  $I(0) = (V_s - V_C(0))/R = -12\text{ V}/R$ . Finite current cannot change capacitor charge instantaneously, so voltage begins at 8 V. Award solution, endpoint check, crossing, signed current and continuity.