

29.1 Polarisation, magnetisation and material fields

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS electric polarisation • magnetisation

Objective

Relate free and bound charge to D , E and polarisation; Compare fixed-charge and fixed-voltage dielectric changes; Use magnetic constitutive response and free-current boundary conditions. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

P is electric dipole moment per volume. $D = \epsilon_0 E + P$ obeys $\nabla \cdot D = \rho_{\text{free}}$. Bound charges obey $\rho_{\text{b}} = -\nabla \cdot P$ and $\sigma_{\text{b}} = P \cdot \mathbf{n}$ with outward $\mathbf{n}$. Uniform P can give surface charge but no bound bulk charge.

For a linear isotropic dielectric, $P = \epsilon_0 \chi_{\text{e}} E$ and $\epsilon_{\text{r}} = 1 + \chi_{\text{e}}$. In a fully filled large parallel-plate capacitor, $D = \sigma_{\text{free}}$ and $E = \sigma_{\text{free}} / \epsilon$. A dielectric need not have zero internal E .

$C = \epsilon A / d$. At fixed Q , $V = Q / C$ and $U = Q^2 / (2C)$ fall as C rises. At fixed V , $Q = CV$ and $U = CV^2 / 2$ rise. The voltage source exchanges energy. Normal D jumps by σ_{free} ; static tangential E is continuous.

M is magnetic dipole moment per volume; $B = \mu_0 (H + M)$. Linear $M = \chi_{\text{m}} H$ gives $B = \mu_0 (1 + \chi_{\text{m}}) H$. A long solenoid has $H \approx nI$. Normal B is continuous; tangential H jumps with free surface current. Hysteresis needs more than constant susceptibility.

■ Worked method

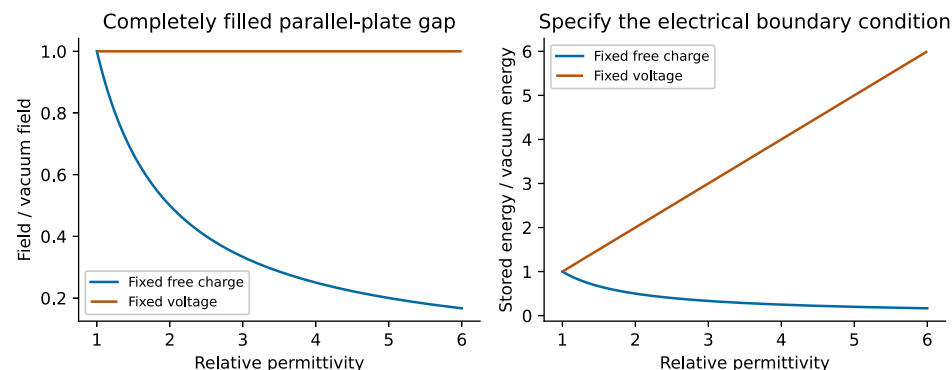
A capacitor is disconnected before inserting a linear dielectric with relative permittivity 4. Free charge stays fixed; capacitance becomes four times its original value.

$$V' = Q / (4C) = V / 4, \quad U' = Q^2 / (2 \times 4C) = U / 4.$$

If an ideal battery stays connected instead, V is fixed:

$$Q' = 4CV = 4Q, \quad U' = \frac{1}{2}(4C)V^2 = 4U.$$

The source exchanges energy. Do not compare isolated charge and fixed voltage as the same experiment.



2 Practice

2.1 Choose one answer. An isolated ideal parallel-plate capacitor is fully filled with a linear dielectric of $\epsilon_{\text{r}} = 3$. Which pair gives voltage and stored-energy ratios? [1]

- **A** 3 and 3
- **B** 1/3 and 1/3
- **C** 1 and 3
- **D** 1/3 and 3
- **E** 1 and 1

2.2 Choose one answer. Uniform electric polarisation _____ in a finite dielectric necessarily implies: [1]

- **A** Nonzero bound volume charge everywhere
- **B** Zero bound surface charge everywhere
- **C** Possible bound surface charge with zero bound bulk charge
- **D** Zero internal electric field as in a metal
- **E** Identical numerical B and H

2.3 In a linear magnetic material with $H = 500$ A/m and $\chi_{\text{m}} = 0.2$, $M =$ _____ A/m. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Normal electric field must be continuous across every dielectric interface even if the permittivities differ. [2]

3 Exam-style questions

3.1 A slab fills half the thickness of a parallel-plate capacitor and has permittivity $3\epsilon_0$; the remaining thickness is vacuum. Find C relative to the empty capacitor, and the two normal fields at fixed free charge Q . Ignore edge effects. [5]

Solutions

2.1 B Free Q stays fixed and C triples; both $V=Q/C$ and $U=Q^2/(2C)$ become one third.

Compare the rejected options: 3 and 3; 1 and 3; 1/3 and 3; 1 and 1. Use the stated model and conditions.

2.2 C $\rho_b = -\nabla \cdot \mathbf{P} = 0$ for uniform $\mathbf{P}$, but $\sigma_b = \mathbf{P} \cdot \mathbf{n}$ can be nonzero.

Compare the rejected options: Nonzero bound volume charge everywhere; Zero bound surface charge everywhere; Zero internal electric field as in a metal; Identical numerical $\mathbf{B}$ and $\mathbf{H}$. Use the stated model and conditions.

2.3 100. $M = \chi_m H = 0.2 \cdot 500 = 100$.

2.4 False. With no free surface charge, normal D is continuous; $E=D/\epsilon$ can differ.

3.1 Layers are in series. With no free interface charge, $D = Q/A$ is common. $E_{vac} = Q/(\epsilon_0 A)$ and $E_d = Q/(3\epsilon_0 A)$. $V = (d/2)(E_{vac} + E_d) = 2Qd/(3\epsilon_0 A)$, hence $C = 3\epsilon_0 A/(2d) = 3C_0/2$. Award common D , each field, voltage sum and capacitance. Normal E is discontinuous; D is continuous.