

21.1 Counting statistics, uncertainty and dimensional models

Name: _____ Class: _____ Date: _____ Total: 10 marks

KEY WORDS standard uncertainty • Poisson approximation

Objective

Use binomial and Poisson count means and fluctuations; Propagate small uncertainties with stated correlation assumptions; Solve dimensional exponent constraints and check a model’s units. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For N independent detection trials with probability p, binomial mean is Np and variance Np(1−p). Standard deviation is its square root. For large N and small p, Poisson mean/variance are λ=Np. Stable efficiency does not guarantee a fixed count; correlations or dead time can invalidate independence.

Linearised independent uncertainties combine as $\sigma_y^2 \approx \sum (\partial y / \partial x_i)^2 \sigma_{x_i}^2$. Correlations need covariance terms. A power $y = x^a$ has fractional uncertainty about $|a| \sigma_x / |x|$. For $K = \frac{1}{2}mv^2$, independent mass/speed contributions are combined as $\text{sqrt}[(\sigma_m/m)^2 + (2\sigma_v/v)^2]$. Use small standard errors, not unstated bounds.

Random scatter measures precision; calibration bias affects accuracy. Independent mean uncertainty falls as $1/\text{sqrt}(n)$, but common bias does not. Physical spread and uncertainty of its mean differ. State whether percentages are standard errors, confidence intervals or worst-case bounds.

For $G\hbar^a c^b$ to be a length, dimensional powers give $-a+b=0$, $3a+2b+d=1$ and $-2a-b-d=0$. Hence $a=b=1/2$, $d=-3/2$ and length $\text{sqrt}(G\hbar/c^3)$. Unit consistency cannot determine a dimensionless coefficient or validate physical assumptions.

■ Worked method

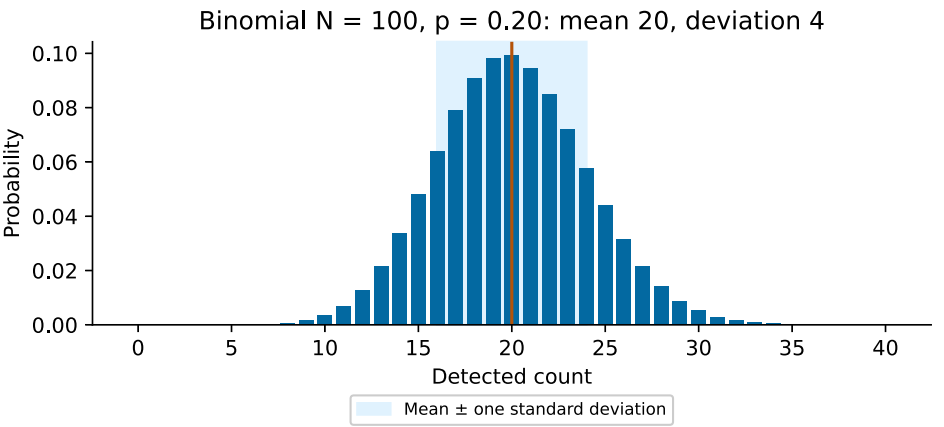
For independent small standard uncertainties in $K = mv^2/2$, differentiate first.

$$\left(\frac{\sigma_K}{K}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(2\frac{\sigma_v}{v}\right)^2.$$

With relative uncertainties 3% in mass and 2% in speed,

$$\sigma_K/K = \sqrt{0.03^2 + (2 \times 0.02)^2} = 0.05.$$

A worst-case bound would use a sum, not this quadrature. Correlations require covariance terms.



2 Practice

2.1 Choose one answer. A binomial detector has N=100 and p=0.20. Its mean and standard deviation are: [1]

- A 20 and 20
- B 20 and 4
- C 20 and 16
- D 5 and 2
- E Exactly 20 with no deviation

2.2 Choose one answer. With negligible mass uncertainty and speed uncertainty 3%, small kinetic-energy fractional uncertainty is: [1]

- A 1.5%
- B 3%
- C 6%
- D 9%
- E 12%

2.3 A Poisson count has mean 25. Its standard deviation is _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: A dimensionally consistent formula necessarily has the correct numerical coefficient and physical assumptions. [2]

3 Exam-style questions

3.1 A sample interval contains 900 counts; an independent equal-duration background interval contains 100. Estimate net counts and their standard uncertainty

. If both intervals are doubled at stable rates, what happens to relative uncertainty?[4]

Solutions

2.1 B Mean 20; variance $100 \cdot 0.2 \cdot 0.8=16$, so standard deviation 4.

Compare the rejected options: 20 and 20; 20 and 16; 5 and 2; Exactly 20 with no deviation. Use the stated model and conditions.

2.2 C K depends on v^2 , so the fractional uncertainty doubles.

Compare the rejected options: 1.5%; 3%; 9%; 12%. Use the stated model and conditions.

2.3 5. Poisson variance equals mean, and standard deviation is its square root.

2.4 False. Units do not determine dimensionless factors or validate the physical model.

3.1 Net count is 800. Independent Poisson variances add despite subtraction: $\sigma = \sqrt{900 + 100} = \sqrt{1000} = 31.62$ counts, relative 0.03953. Doubled intervals give net 1600 and variance 2000, so relative uncertainty is divided by $\sqrt{2}$. Award net, variance addition, uncertainty and time scaling. This assumes stable rates and no dead-time correlations.