

11.1 Rotation, buoyancy and terminal-speed balances

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS moment of inertia • terminal speed

Objective

Use pivot torque and rotational inertia consistently; Partition rolling kinetic energy with the no-slip constraint; Balance buoyancy and drag for composite bodies. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

About a fixed pivot, torque is $\mathbf{r} \times \mathbf{F}$. A uniform rod pivoted at one end has $I = ML^2/3$; gravity acts at $L/2$. With angle θ from vertical, torque magnitude is $Mg(L/2)\sin\theta$, giving angular-acceleration magnitude $3g \sin\theta/(2L)$. Define direction separately; centre-of-mass inertia needs a parallel-axis correction.

No-slip rolling gives $v = R\omega$. With $I = \beta MR^2$, $K = \frac{1}{2}Mv^2(1 + \beta)$ and the rotational fraction is $\beta/(1 + \beta)$. Disk: $1/3$; sphere: $2/7$; hoop: $1/2$. Static friction can provide torque without dissipating energy at a stationary contact on a fixed surface.

Floating composites displace enough fluid to balance total weight. If submerged wood fraction is f with a stone dry, then putting stone volume V_s fully underwater reduces the wood fraction to $f - V_s/V$. The stone retains its weight while adding buoyancy; check that flotation/orientation assumptions are possible.

Terminal speed is a force balance. For quadratic drag Cv^2 and negligible buoyancy, $v_t = \sqrt{Mg/C}$. Identical shapes share C , so a mass ratio r gives speed ratio $\sqrt{r}$. With buoyancy use effective weight; linear drag gives a different scaling. Terminal motion has nonzero balancing forces.

■ Worked method

A body rolls without slipping on a fixed surface. The no-slip constraint is $v = R\omega$.

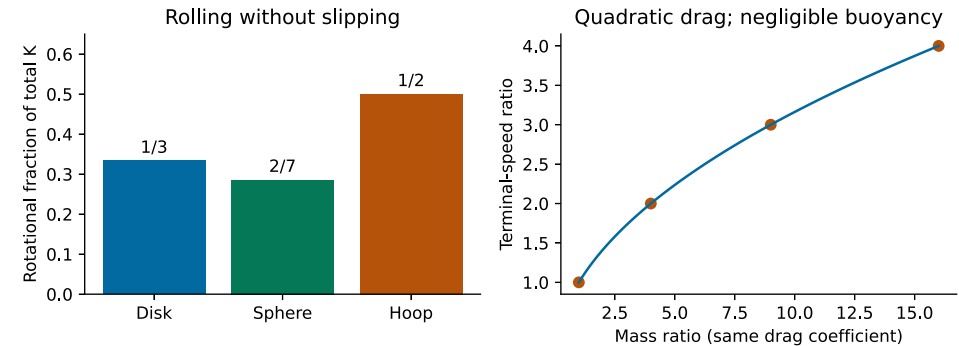
$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2(1 + \beta), \quad I = \beta MR^2.$$

For a solid disk, $\beta = 1/2$, so the rotational share is

$$\frac{K_{rot}}{K} = \frac{\beta}{1 + \beta} = \frac{1/2}{3/2} = \frac{1}{3}.$$

For a uniform rod pivoted at one end, use $I = ML^2/3$, not its centre-of-mass inertia. About this pivot $\tau = -Mg(L/2)\sin\theta$, for θ measured from downward vertical.

$$\ddot{\theta} = \tau/I = -3g \sin\theta/(2L).$$



2 Practice

2.1 For a uniform rod pivoted at one end, gravitational angular-acceleration magnitude at angle θ from vertical is: [1]

- A. $g \sin\theta/L$
- B. $3g \sin\theta/(2L)$
- C. $3g \cos\theta/(2L)$
- D. $6g \sin\theta/L$
- E. $g/(2L)$

2.2 For a solid sphere $I = 2MR^2/5$ rolling without slipping, the rotational fraction of total K is: [1]

- A. $2/5$
- B. $1/2$
- C. $2/7$
- D. $5/7$
- E. $1/5$

2.3 Identical-shape balls have quadratic drag and masses M and $9M$, with negligible buoyancy. Their terminal-speed ratio is _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: At terminal speed the drag force vanishes. [2]

3 Exam-style questions

3.1 A solid sphere rolls from rest down a vertical height h without slipping or energy loss. Use $I = 2MR^2/5$ to find its bottom speed. Compare with a frictionless sliding point mass. State which assumption would fail if the sphere slipped. [4]

Solutions

2.1 Answer: B. Divide $Mg(L/2)\sin\theta$ by $ML^2/3$.

Compare the rejected options: $g \sin\theta/L$; $3g \cos\theta/(2L)$; $6g \sin\theta/L$; $g/(2L)$. Use the stated model and conditions.

2.2 Answer: C. Use $\beta/(1+\beta)=(2/5)/(7/5)=2/7$.

Compare the rejected options: $2/5$; $1/2$; $5/7$; $1/5$. Use the stated model and conditions.

2.3 3. The ratio is $\sqrt{9}$.

2.4 False. Drag balances the effective weight; it is generally nonzero.

3.1 Energy gives $Mgh = \frac{1}{2}Mv^2(1+2/5)$, hence $v = \sqrt{10gh/7}$. The slider has $v_s = \sqrt{2gh}$, so $v/v_s = \sqrt{5/7} < 1$. Slipping breaks $v = R\omega$ and can dissipate energy through kinetic friction. Award energy relation, speed, comparison and the constraint/loss distinction.