

10.1 Collisions, work and oscillator energy

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS external impulse • turning point

Objective

Compute vector momentum and energy loss in sticking collisions; Compare fixed-force springs and use work–energy conditions; Derive small-oscillation periods and oscillator energies. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

With negligible external impulse, conserve momentum in each direction. For sticking masses, $\mathbf{v}=(m_1\mathbf{v}_1+m_2\mathbf{v}_2)/(m_1+m_2)$. Add vectors before taking magnitudes. Compare $\Sigma \frac{1}{2}m|\mathbf{v}|^2$ initially with $\frac{1}{2}(m_1+m_2)|\mathbf{v}|^2$ finally; the loss becomes internal energy/deformation. Sticking does not mean final rest.

Work–energy gives $\Delta K=\int \mathbf{F}\cdot d\mathbf{r}$, or Fs for constant parallel force. Mechanical-energy conservation requires no unaccounted nonconservative work. Spring energy is $\frac{1}{2}kx^2$. At fixed force, $x=F/k$ and $U=F^2/(2k)$; at fixed extension U increases with k . These comparisons have opposite answers.

Near stable equilibrium, $U\approx U_0+\frac{1}{2}k_{\text{eff}}(x-x_0)^2$ with $k_{\text{eff}}=U''(x_0)>0$. Then $\omega=\text{sqrt}(k_{\text{eff}}/m)$, $T=2\pi/\omega$. A small-angle pendulum uses $\sin\theta\approx\theta$ in $\theta\ddot{+}(g/L)\sin\theta=0$, giving $T=2\pi\text{sqrt}(L/g)$. Large amplitudes need a correction; ideal pendulum mass cancels.

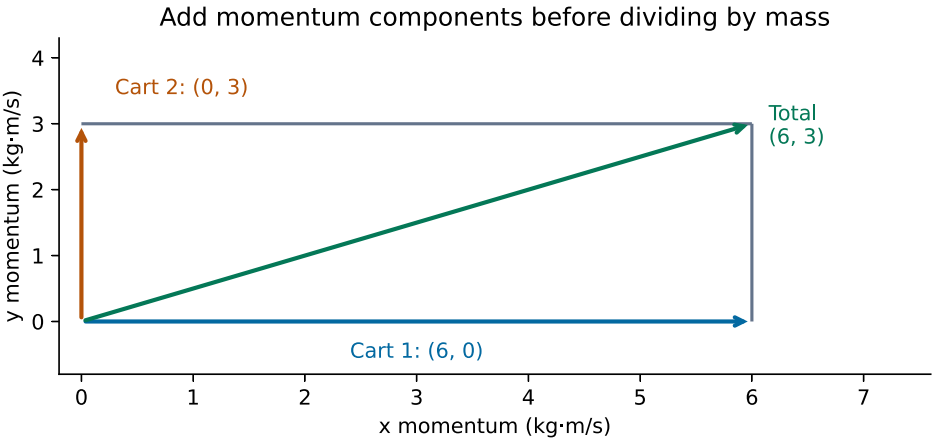
Undamped oscillator energy is $\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$. At equilibrium it is all kinetic; at turning points it is all potential. Use $\omega=2\pi f$ and $A=v_{\text{max}}/\omega$. The measurement position determines which energy expression is simplest; convert joules to millijoules only after calculation.

■ Worked method

Two carts stick with negligible external impulse. Add momentum vectors, then divide by total mass. Known: $m_1=2\text{ kg}$, $\mathbf{v}_1=(3,0)\text{ m/s}$; $m_2=1\text{ kg}$, $\mathbf{v}_2=(0,3)\text{ m/s}$.

$$\mathbf{v}_f=\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{m_1+m_2}=\frac{(2\text{ kg})(3,0)\text{ m/s}+(1\text{ kg})(0,3)\text{ m/s}}{3\text{ kg}}=(2,1)\text{ m/s}.$$
$$K_i=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}(2\text{ kg})(3\text{ m/s})^2+\frac{1}{2}(1\text{ kg})(3\text{ m/s})^2=13.5\text{ J}.$$
$$K_f=\frac{1}{2}(m_1+m_2)|\mathbf{v}_f|^2=\frac{1}{2}(3\text{ kg})(5\text{ m}^2/\text{s}^2)=7.5\text{ J}.$$

The 6 J difference becomes internal energy. Kinetic energy is not conserved in sticking.



2 Practice

2.1 Choose one answer. For the two carts in the worked example, the common velocity is: [1]

- A (3,3) m/s
- B (2,1) m/s
- C (1,2) m/s
- D (2,2) m/s
- E (0,0) m/s

2.2 Choose one answer. Two springs with $k_1=3k_2$ hold the same force. The ratio U_1/U_2 is: [1]

- A 3
- B 9
- C 1
- D $1/3$
- E $1/9$

2.3 A small-angle pendulum length is multiplied by 9 at fixed g . Its period is multiplied by _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: At equilibrium an undamped harmonic oscillator can have nonzero kinetic energy and zero spring potential energy. [2]

3 Exam-style questions

3.1 Two equal masses approach with speed u along directions separated by angle θ and stick. Derive final speed and the fraction of initial kinetic energy lost. Check parallel, perpendicular and opposite incoming directions. [5]

Solutions

2.1 B Divide the vector momentum (6,3) by total mass 3, component by component.

Compare the rejected options: (3,3) m/s; (1,2) m/s; (2,2) m/s; (0,0) m/s. Use the stated model and conditions.

2.2 D At fixed force $U=F^2/(2k)$, hence the inverse stiffness ratio.

Compare the rejected options: 3; 9; 1; 1/9. Use the stated model and conditions.

2.3 3. T scales as $\sqrt{L}$.

2.4 True. Displacement is zero, so spring energy is zero; speed is maximal.

3.1 The sum of equal velocity vectors has magnitude $2u|\cos(\theta/2)|$. Dividing momentum by total mass gives $v_f = u|\cos(\theta/2)|$. $K_i = mu^2$ and $K_f = mv_f^2$, so lost fraction is $\sin^2(\theta/2)$. The three limits are 0, 1/2 and 1. Award vector sum, speed, energy relation, lost fraction and limits. The formula uses $0 \leq \theta \leq \pi$ between directions.