

Exercise walk-through

8.1

Solid-state, nuclear and particle models

GRE · GRE Subject Test · GRE Physics

8.1 Solid-state, nuclear and particle models

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS band gap • half-life

Objective

Use band and carrier descriptions; Apply radioactive decay and binding-energy ideas; Check conservation laws in particle reactions. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Filled bands cannot carry ordinary current without accessible nearby states. Metals have partially filled or overlapping bands; semiconductors have a small band gap. Donor and acceptor doping change carrier populations.

Fermi–Dirac occupancy is $1/[\exp((E-\mu)/(kT))+1]$. At zero temperature it becomes a

step at the Fermi energy; thermal broadening does not make every state half occupied.

Radioactive populations obey $N=N_0e^{(-\lambda t)}$, activity λN and half-life $\ln 2/\lambda$.
Binding energy comes from a mass deficit multiplied by c^2 ; compare binding energy per nucleon when discussing stability.

Nuclear and particle processes must conserve charge, energy, momentum and the applicable quantum numbers. Beta decay includes a neutrino or antineutrino; a two-body electron-only story cannot explain its continuous energy spectrum.

■ **Worked method**

Binding energy is the positive energy required to separate a nucleus into free nucleons. For a reaction use total initial and final rest masses, with a consistent mass convention.

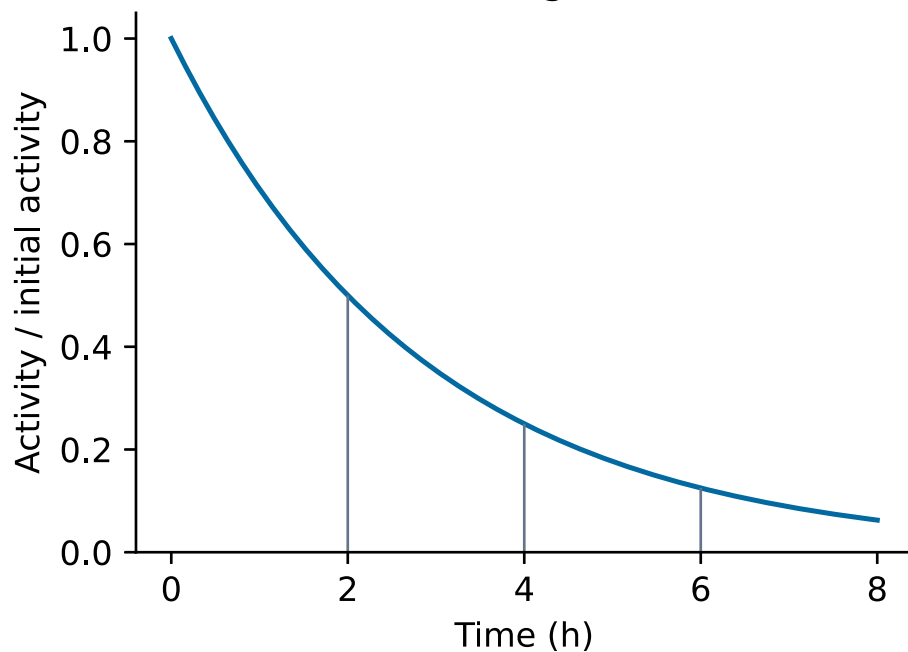
$$Q = (M_i - M_f)c^2.$$

A mass decrease of 0.0040 atomic mass units gives

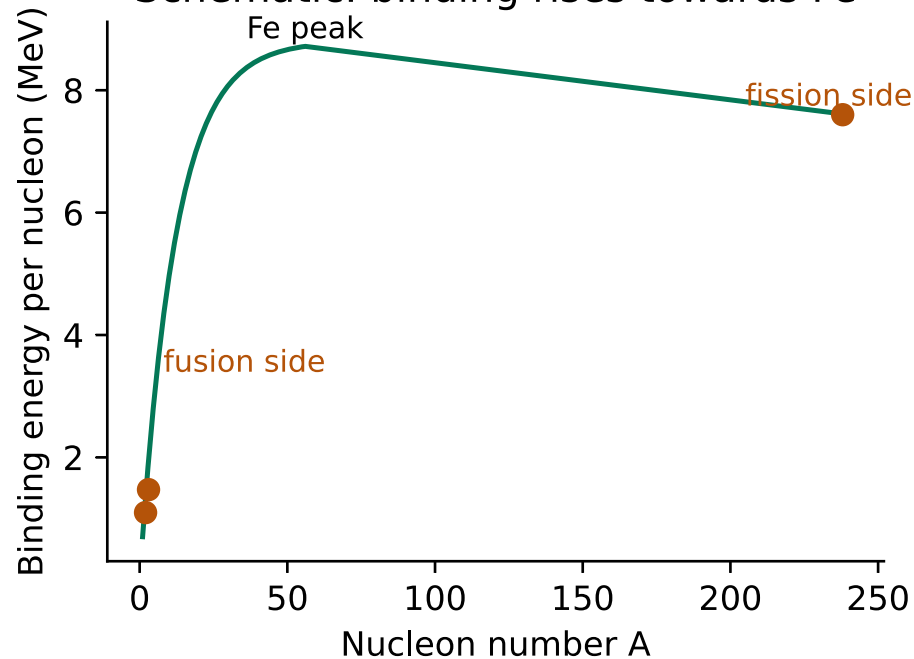
$$Q = \Delta Mc^2 = (0.0040 \text{ u})(931.5 \text{ MeV/u}) = 3.73 \text{ MeV}.$$

Positive Q means energy release. Charge, nucleon number, energy and momentum must still balance; Q alone does not establish a reaction rate.

$T_{1/2} = 2 \text{ h}$: halving each interval



Schematic: binding rises towards Fe



Exercises 2.1·2.2

2.1 Choose one answer. After two half-lives, the remaining fraction is: [1]

- **A** $1/2$
- **B** $1/4$
- **C** $1/8$

2.2 Choose one answer. Donor doping commonly supplies: [1]

- **A** additional electron carriers
- **B** only photons
- **C** no mobile carriers

Exercises 2.1·2.2

2.1 Choose one answer. After two half-lives, the remaining fraction is: [1]

- A $1/2$
- B $1/4$
- C $1/8$

2.1 B Two successive halvings give one quarter.

Compare the rejected options: $1/2$; $1/8$. Use the stated model and conditions.

2.2 Choose one answer. Donor doping commonly supplies: [1]

- A additional electron carriers
- B only photons
- C no mobile carriers

2.2 A Donor levels facilitate electron excitation into conducting states.

Compare the rejected options: only photons; no mobile carriers. Use the stated model and conditions.

Exercises 2.3·3.1

2.3 After four half-lives, one part in _____ remains. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Charge conservation alone is sufficient to validate any particle reaction. [2]

3 Exam-style questions

3.1 A neutral stationary parent nucleus decays into two neutral fragments with masses m and $3m$. Released kinetic energy is Q , small compared with either rest energy. Find each fragment's kinetic energy and speed ratio. [4]

Exercises 2.3-3.1

2.3 After four half-lives, one part in _____ remains. Show your relation and units. [2]

2.3 16. $2^4=16$.

2.4 Decide whether this claim is true. Justify it: Charge conservation alone is sufficient to validate any particle reaction. [2]

3 Exam-style questions

2.4 False. Energy, momentum and other applicable conservation laws also matter.

3.1 A neutral stationary parent nucleus decays into two neutral fragments with masses m and $3m$. Released kinetic energy is Q , small compared with either rest energy. Find each fragment's kinetic energy and speed ratio. [4]

3.1 Momentum conservation gives equal and opposite momenta of magnitude p . Non-relativistic energies are $K_1 = p^2/(2m)$ and $K_2 = p^2/(6m)$, hence ratio 3:1. Their sum is Q , so $K_1 = 3Q/4$ and $K_2 = Q/4$. Speeds obey $v_1/v_2 = (p/m)/(p/(3m)) = 3$. Award momentum, energy ratio, partition and speed ratio. Equal momentum does not mean equal energy.