

Exercise walk-through

5.1

Quantum mechanics and atomic physics

GRE · GRE Subject Test · GRE Physics

5.1 Quantum mechanics and atomic physics

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS normalisation • operator

Objective

Use wavefunctions, operators and measurement probabilities; Solve standard bound-state models and tunnelling reasoning; Apply angular momentum, spin and atomic spectral principles. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Normalise so $\int |\psi|^2 dx = 1$. In an orthonormal expansion $\psi = \sum c_j \varphi_j$ the norm squared is $\sum |c_j|^2$; each energy probability is $|c_j|^2$.

Hermitian observables have real eigenvalues. The expectation $\langle A \rangle = \int \psi^* A \psi dx$ is a repeated-preparation average. Apply operator identities to eigenstates: $A^4 = I$ gives

only real eigenvalues $+1$ and -1 .

For an infinite well $0 < x < L$, zero endpoint values give sine modes with $E_n = n^2 \pi^2 \hbar^2 / (2mL^2)$ $n \geq 1$. Doubling L quarters the energies. Finite barriers permit exponential tails.

Commuting observables share an eigenbasis. $T = p^2 / (2m)$ commutes with p ; a variable $V(x)$ generally breaks $[H, p] = 0$. $[J_x, J_y] = i\hbar J_z$; J^2 commutes with each component. Photon energies equal level differences, subject to selection rules.

■ **Worked method**

An infinite well extends from 0 to L . Vanishing wavefunction at both ends gives

$$\psi_n = \sqrt{2/L} \sin(n\pi x/L), \qquad E_n = n^2 \pi^2 \hbar^2 / (2mL^2).$$

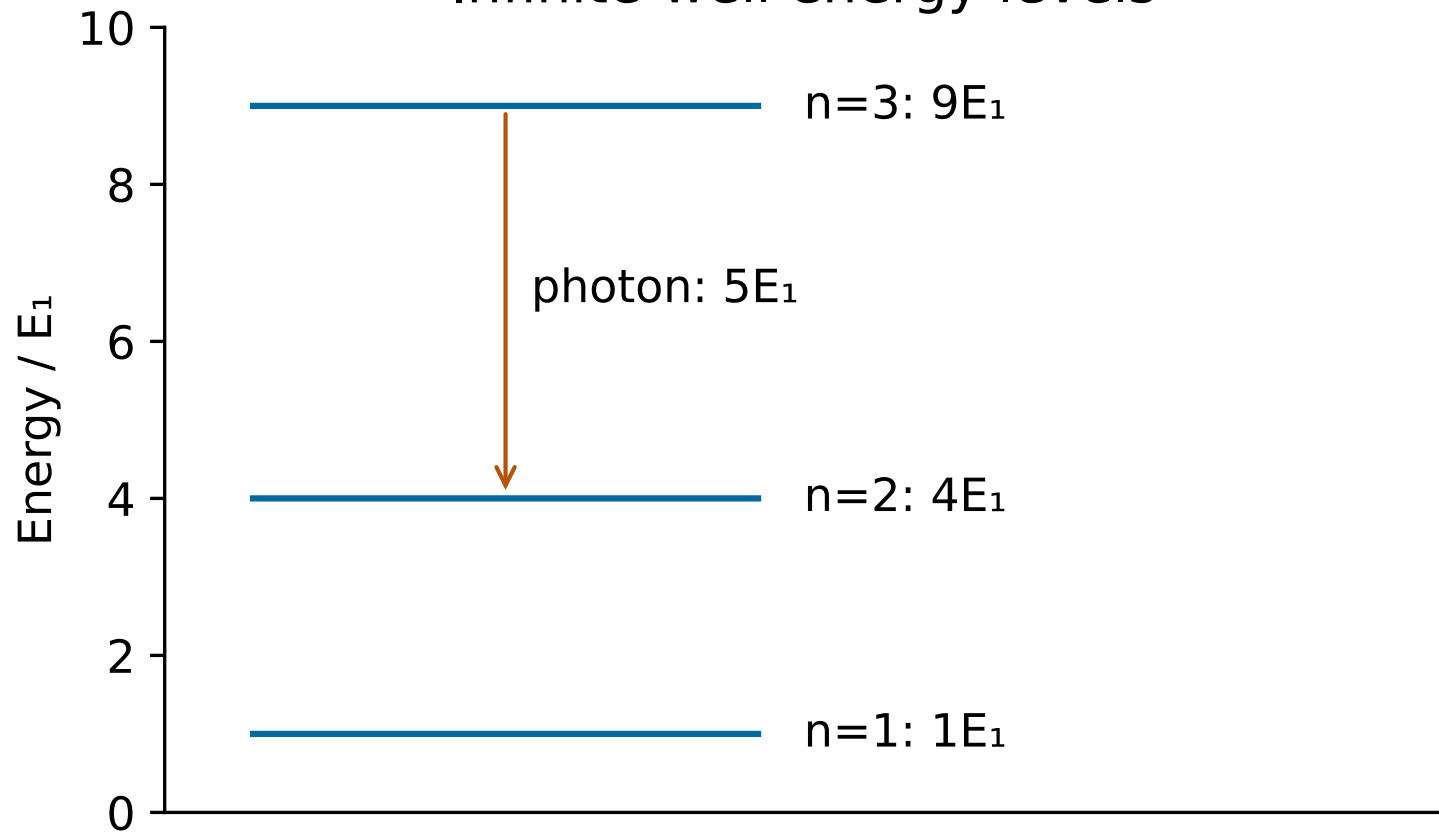
Prepare $\psi = (\psi_1 + i\psi_2) / \sqrt{2}$. Orthogonality makes its norm one. Energy measurements

give E_1 or $4E_1$ with equal probability:

$$\langle E \rangle = \frac{1}{2}E_1 + \frac{1}{2}(4E_1) = \frac{5}{2}E_1.$$

This average is not an allowed single energy result. The relative phase changes in time, so a superposition of different energies is not a stationary state.

Infinite-well energy levels



Exercise 2.1

2.1 Choose one answer. In an infinite well, E_3/E_1 is:

[1]

- **A** 3
- **B** 6
- **C** 9
- **D** The boundary conditions determine no energies
- **E** Replace $\hbar$ with h without changing the coefficient

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2.1 C Energy scales as n^2 .

Compare the rejected options: 3; 6; The boundary conditions determine no energies; Replace $\hbar$ with h without changing the coefficient. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. The probability density is: [1]

- **A** ψ
- **B** $|\psi|^2$
- **C** the energy alone
- **D** The boundary conditions determine no energies
- **E** Replace $\hbar$ with h without changing the coefficient

2.3 If $E_1=2$ eV, $E_2=\rule{1cm}{0.4pt}$ eV. Show your relation and units. [2]

Exercises 2.2·2.3

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2.2 B The modulus squared gives density.

Compare the rejected options: ψ ; the energy alone; The boundary conditions determine no energies; Replace $\hbar$ with h without changing the coefficient. Use the stated model and conditions.

2.3 If $E_1=2$ eV, $E_2=$ _____ eV. Show your relation and units. [2]

2.3 8. $E_2=4E_1=8$ eV.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: A normalised quantum state has total probability one. [2]

3.1 In the same well a normalised state is $(\psi_1 + 2\psi_3)/\sqrt{5}$. Find its possible measured energies and probabilities, mean energy and energy variance. Is the mean an allowed result of one measurement? [5]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: A normalised quantum state has total probability one. [2]

2.4 True. Integrating its probability density over the full domain gives one.

3.1 In the same well a normalised state is $(\psi_1 + 2\psi_3)/\sqrt{5}$. Find its possible measured energies and probabilities, mean energy and energy variance. Is the mean an allowed result of one measurement? [5]

3.1 Results are E_1 with probability $1/5$ and $9E_1$ with probability $4/5$. $\langle E \rangle = 37E_1/5$; $\langle E^2 \rangle = 65E_1^2$. $\text{Var}(E) = 65E_1^2 - (37E_1/5)^2 = 256E_1^2/25$. The mean $7.4E_1$ is not a well eigenvalue. Award results, probabilities, mean, second-moment/variance calculation and interpretation.