

Exercise walk-through

# 46.1

## Reaction energies, activity and decay balances

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GRE · GRE Subject Test · GRE Physics

# 46.1 Reaction energies, activity and decay balances

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 11 marks**

**KEY WORDS** Q value Q • activity

# Objective

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Compute reaction energy from complete atomic or nuclear mass bookkeeping; Calculate activity from half-life and a simple decay-chain balance; Compare fission and fusion binding changes with charge and nucleon conservation. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

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$Q=(m_{\text{initial}}-m_{\text{final}})c^2$  with  $1\text{ u}=931.5\text{ MeV}$ . Atomic masses balance electrons directly for  $\beta^-$  and neutral-He  $\alpha$ ;  $\beta^+$  needs explicit electron masses.

$A=\lambda N$  with  $\lambda=\ln 2/T_{1/2}$ . Equal activities need not mean equal atom counts.

Chains:  $dN_B/dt = \lambda_A N_A - \lambda_B N_B$ ; a long-lived parent gives secular equilibrium of activities.

The binding-per-nucleon curve peaks near iron: heavy fission and light fusion both raise total binding and release energy.

### ■ Worked method

Activity is  $A = \lambda N$ , with  $\lambda = \ln 2 / T_{1/2}$ . For  $N = 6.0 \times 10^{12}$  and half-life 2 hours, convert time before calculating.

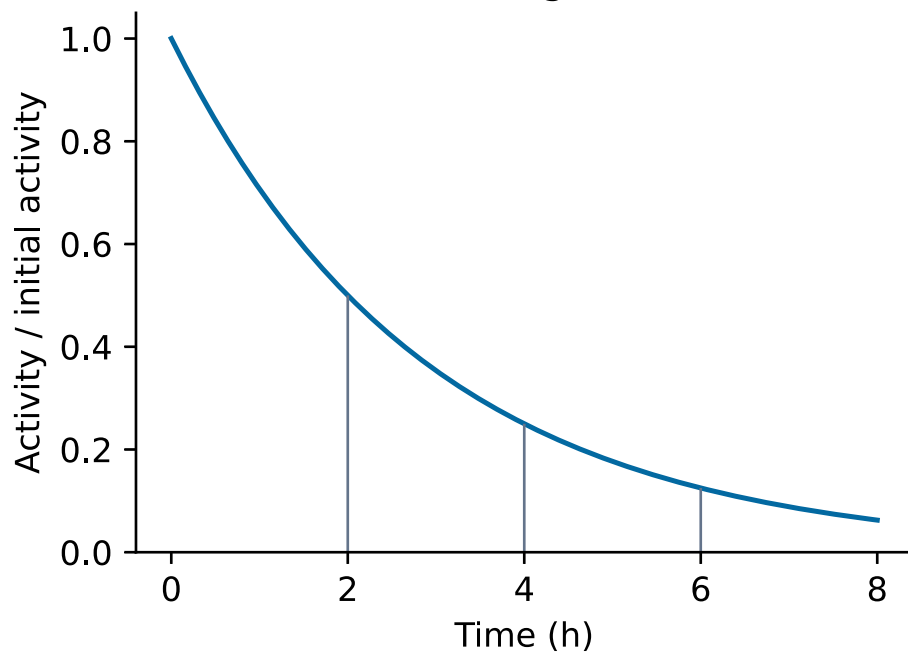
$$A = (\ln 2)N / T_{1/2} = (\ln 2)(6.0 \times 10^{12}) / (7200 \text{ s}) = 5.78 \times 10^8 \text{ Bq}.$$

A decay chain requires both production and loss terms:  $\dot{N}_B = \lambda_A N_A - \lambda_B N_B$ . Equal activities do not imply equal populations. Atomic-mass beta-plus Q values require sub-

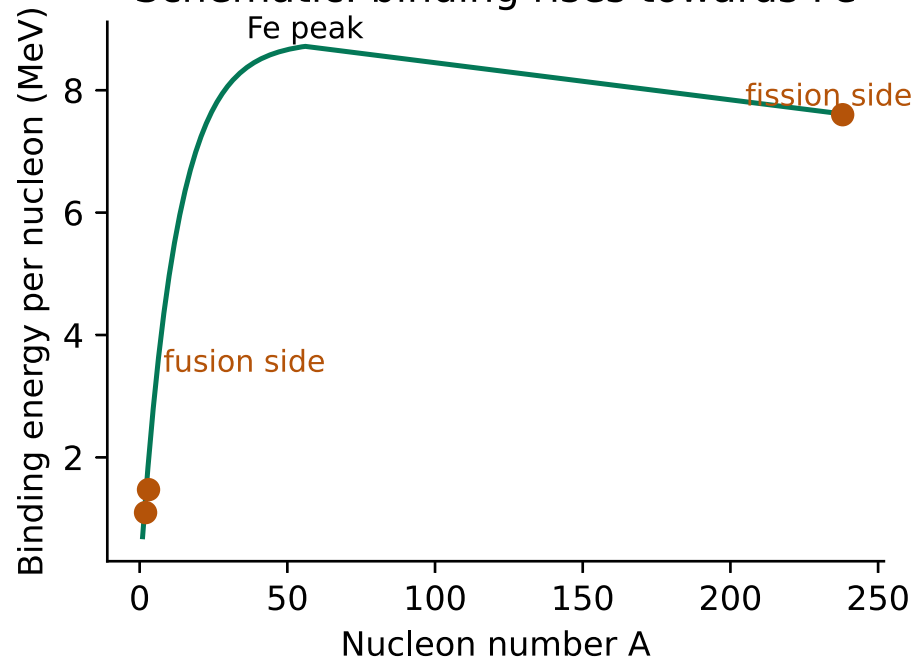
tracting  $2m_e c^2$ .

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$T_{1/2} = 2 \text{ h}$ : halving each interval



Schematic: binding rises towards Fe



## Exercise 2.1

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**2.1** Choose one answer. Using atomic masses 238.0508 u ( $^{238}\text{U}$ ), 234.0436 u ( $^{234}\text{Th}$ ) and 4.0026 u ( $^4\text{He}$ ), the alpha-decay Q value Q is about: [1]

- **A** 0.46 MeV
- **B** 2.14 MeV
- **C** 4.28 MeV
- **D** 6.55 MeV
- **E** 9.31 MeV

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- **A** 0.46 MeV
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- **D** 6.55 MeV
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**2.1 C**  $\Delta m = 0.0046 \text{ u}$ ;  $0.0046 \times 931.5 \approx 4.28 \text{ MeV}$ .

Compare the rejected options: 0.46 MeV; 2.14 MeV; 6.55 MeV; 9.31 MeV. Use the stated model and conditions.

## Exercises 2.2·2.3

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**2.2** Choose one answer. Two samples contain equal numbers of radioactive nuclei; sample X has half the half-life of sample Y. The activity of X is: [1]

- A half that of Y
- B equal to that of Y
- C twice that of Y
- D four times that of Y
- E one quarter that of Y

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**2.3** A sample starts with activity  $8 \times 10^8$  Bq and half-life 3 h. After 9 h its activity is

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**2.2 C**  $A = \lambda N$  and  $\lambda = \ln 2 / T^{1/2}$ , so halving  $T^{1/2}$  doubles  $A$  at fixed  $N$ .

Compare the rejected options: half that of Y; equal to that of Y; four times that of Y; one quarter that of Y. Use the stated model and conditions.

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**2.3** A sample starts with activity  $8 \times 10^8$  Bq and half-life 3 h. After 9 h its activity is

**2.3 1.** Nine hours is three half-lives:  $8 \times 10^8 / 8 = 1 \times 10^8$  Bq.

## Exercises 2.4·3.1

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**2.4** Decide whether this claim is true. Justify it: For  $\beta^-$  decay, tabulated atomic masses of parent and daughter require an extra electron-mass term in  $Q$ . [2]

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**3.1** A parent has initial population  $N_0$  and daughter population zero. Both decay constants equal  $\lambda$ . Derive daughter population and the time of its maximum. [5]

## Exercises 2.4·3.1

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**2.4** Decide whether this claim is true. Justify it: For  $\beta^-$  decay, tabulated atomic masses of parent and daughter require an extra electron-mass term in  $Q$ . [2]

**2.4** False. Atomic electron counts already balance for  $\beta^-$  (and for  $\alpha$  with neutral He);  $\beta^+$  needs the explicit  $2m_e$ .

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**3.1** A parent has initial population  $N_0$  and daughter population zero. Both decay constants equal  $\lambda$ . Derive daughter population and the time of its maximum. [5]

**3.1** Parent population is  $N_A = N_0 e^{-\lambda t}$ . Daughter equation  $\dot{N}_B + \lambda N_B = \lambda N_0 e^{-\lambda t}$  has