

Exercise walk-through

42.1

Simultaneity, length measurement and velocity transformation

GRE · GRE Subject Test · GRE Physics

42.1 Simultaneity, length measurement and velocity transformation

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS spacetime interval • proper length

Objective

Transform event separations and identify proper length versus simultaneous endpoint readings; Apply relativistic velocity addition and check a four-vector invariant; Classify the interval sign to order events and rule out causal contact. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Transform separations with $x'=\gamma(x-vt)$, $t'=\gamma(t-vx/c^2)$. Events simultaneous in S with

$\Delta x \neq 0$ are not simultaneous in S' : $\Delta t' = -\gamma v \Delta x / c^2$.

A frame length uses simultaneous endpoint readings in that frame; then $L = L_0 / \gamma$. Different reading conditions answer different questions.

$s^2 = c^2 \Delta t^2 - \Delta x^2$ is invariant: timelike separations have a co-located frame and fixed time order; spacelike separations have a simultaneous frame and can swap order.

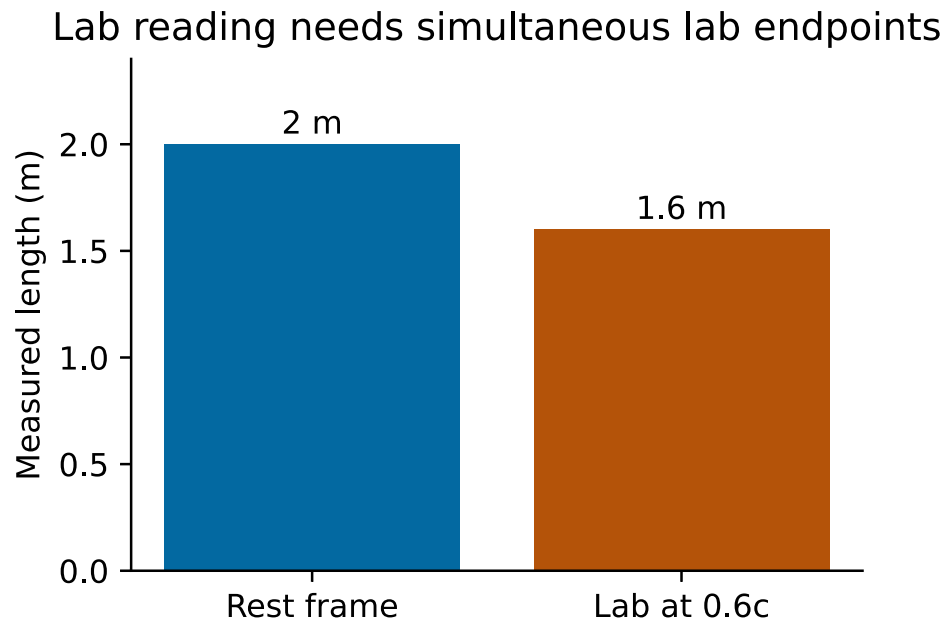
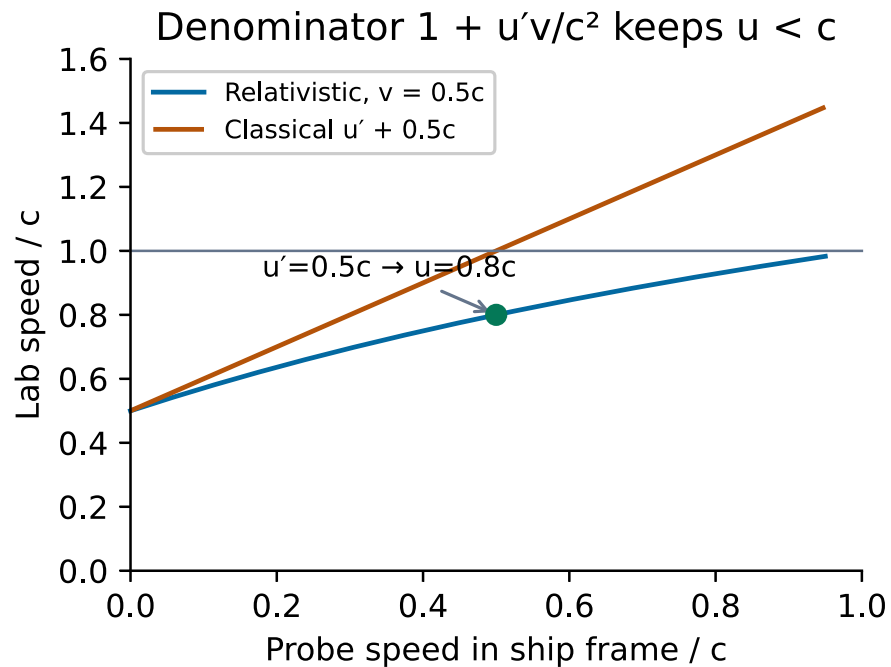
$u = (u' + v) / (1 + u'v/c^2)$ keeps speeds below c ; $E^2 - p^2 c^2 = m^2 c^4$ is frame-independent. Check the nonrelativistic limit.

■ **Worked method**

Two lab events have $\Delta x = 900 \text{ m}$ and $\Delta t = 2 \text{ microseconds}$. Their spacetime interval is spacelike because $c \Delta t = 600 \text{ m}$ is less than Δx . A frame making them simultaneous satisfies

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0, \quad v = c^2\Delta t/\Delta x = 2c/3.$$

Their separation in that frame is $\sqrt{\Delta x^2 - c^2\Delta t^2} = 670.8$ m. No frame can put these spacelike events at the same position.



Exercise 2.1

2.1 Choose one answer. To obtain the contracted length of a moving rod, a lab observer must record the two endpoint positions: [1]

- **A** simultaneously in the rod rest frame
- **B** simultaneously in the lab frame
- **C** at the same lab position at different times
- **D** at equal rod-frame times only
- **E** without any simultaneity condition

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2.1 B Length in a frame is the endpoint separation at equal time in that frame; $\Delta t=0$ in the lab gives $L=L_0/\gamma$.

Compare the rejected options: simultaneously in the rod rest frame; at the same lab position at different times; at equal rod-frame times only; without any simultaneity condition. Use the stated model and conditions.

Exercises 2.2-2.3

2.2 Choose one answer. A ship moves at $0.5c$ and launches a probe forward at $0.5c$ relative to the ship. The probe has lab speed: [1]

- **A** $0.75c$
- **B** $0.8c$
- **C** $0.9c$
- **D** $1.0c$
- **E** exactly c

2.3 Two lab events have $\Delta x = 900$ m and $\Delta t = 2 \mu s$. Using $c = 3 \times 10^8$ m/s, the frame in

Exercises 2.2·2.3

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2.2 B $(0.5+0.5)/(1+0.25)=0.8$; the denominator keeps the result below c .

Compare the rejected options: $0.75c$; $0.9c$; $1.0c$; exactly c . Use the stated model and conditions.

2.3 Two lab events have $\Delta x=900$ m and $\Delta t=2 \mu\text{s}$. Using $c=3\times 10^8$ m/s, the frame in

2.3 2/3. Set $\Delta t'=0$ in $t'=\gamma(t-vx/c^2)$: $v=c^2\Delta t/\Delta x=(600/900)c=2c/3$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: If two events are spacelike separated, every inertial frame agrees on which event happened first. [2]

3.1 Two events are separated by 300 m and 2 microseconds in the lab. Find the velocity of a frame in which they occur at one position, and their time separation there. Can a frame reverse their order? [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: If two events are spacelike separated, every inertial frame agrees on which event happened first. [2]

2.4 False. Spacelike order can swap between frames; only timelike or lightlike order is invariant.

3.1 Two events are separated by 300 m and 2 microseconds in the lab. Find the velocity of a frame in which they occur at one position, and their time separation there. Can a frame reverse their order? [4]

3.1 Set $\Delta x' = \gamma(\Delta x - v\Delta t) = 0$, so $v = \Delta x / \Delta t = 0.5c$. The proper time is $\sqrt{\Delta t^2 - (\Delta x/c)^2} = \sqrt{3}$ microseconds. They are timelike separated, and every time-oriented inertial frame retains their order. Award velocity condition, speed, proper time and causal classification/order.