

Exercise walk-through

40.1

Atomic field shifts and spectral differences

GRE · GRE Subject Test · GRE Physics

40.1 Atomic field shifts and spectral differences

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS Landé factor • Stark splitting

Objective

Calculate orbital and spin magnetic shifts in the appropriate coupling regime; Find photon-energy shifts from allowed upper-minus-lower level changes; Contrast parity cancellation with degenerate electric-field splitting. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For B along z , electron $H_Z = \mu_B B (L_z + 2S_z) / \hbar$. In the uncoupled regime, $\Delta E = \mu_B B (m_l + 2m_s)$. Compare magnetic energy with spin-orbit and orbital gaps.

Weak LS-coupled fields use $\Delta E = \mu_B B g_J m_J$ and the Landé factor . For $L=1, S=1/2$: $g_J(3/2)=4/3$, $g_J(1/2)=2/3$. $J=0$ needs separate treatment.

Photon shift is upper minus lower shift, subject to transition selection rules. A spin-neglected $L=1 \rightarrow 0$ model has three photon shifts $-\mu_B B, 0, +\mu_B B$.

A nondegenerate parity state has zero first-order electric dipole expectation. A degenerate opposite-parity pair with supplied dipole matrix $[[0,d],[d,0]]$ has shifts $\pm dF$. Higher-order and near-degenerate corrections remain possible.

■ Worked method

In weak LS-coupled magnetic fields, the Zeeman effect uses J , not uncoupled m_l and m_s .

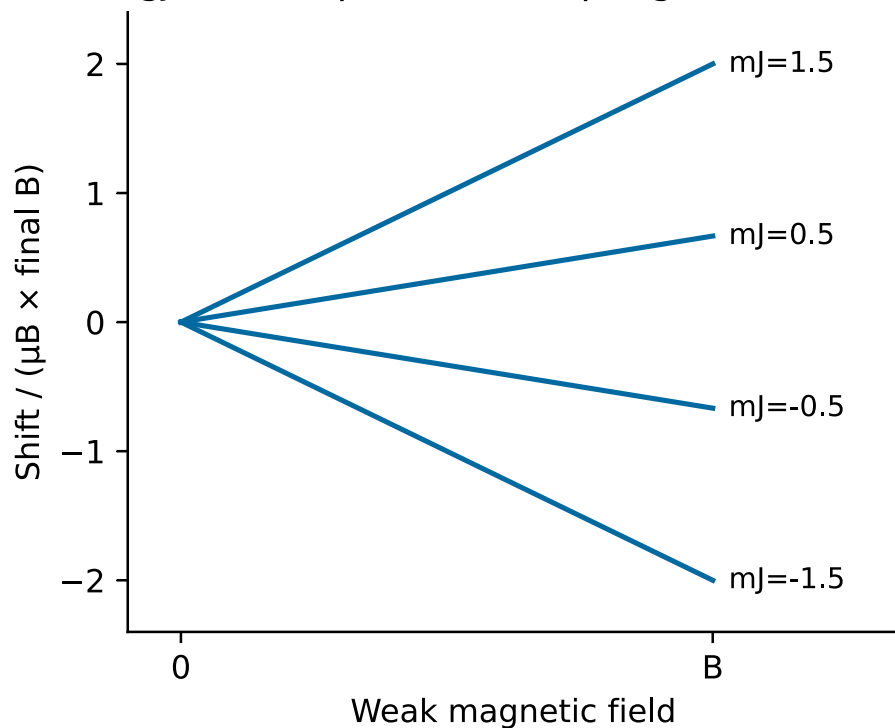
$$\Delta E = \mu_B B g_J m_J, \quad g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

For $L = 1$, $S = 1/2$ and $J = 3/2$, $g_J = 4/3$.

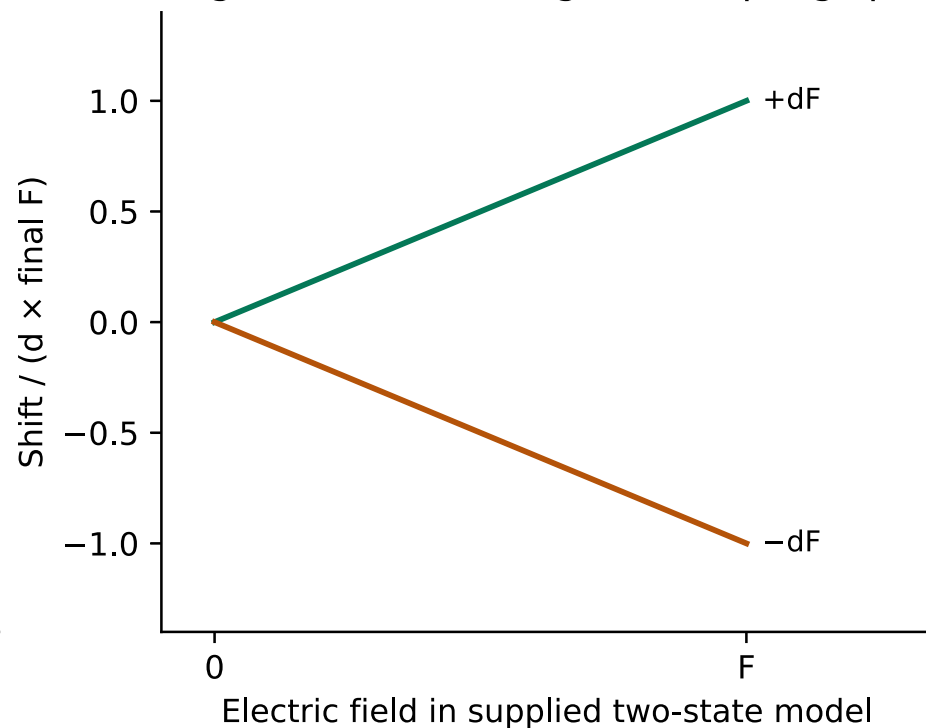
$$\Delta E = \mu_B B g_J m_J = (5.788 \times 10^{-5} \text{ eV/T})(2 \text{ T})(4/3)(1/2) = 7.72 \times 10^{-5} \text{ eV}.$$

Select the field regime before inserting quantum numbers. Photon shifts subtract the lower-level shift.

LS-coupled $L=1$, $S=\frac{1}{2}$, $J=\frac{3}{2}$
 $g_J = 4/3$; spin-orbit coupling dominates



Degenerate opposite-parity pair
Diagonal zero; off-diagonal coupling splits



Exercise 2.1

2.1 Choose one answer. In the uncoupled electron approximation, $m_l=1, m_s=-1/2$ and B along $+z$ give a linear magnetic shift: [1]

- **A** $-2\mu_B B$
- **B** $-\mu_B B$
- **C** 0
- **D** $+\mu_B B$
- **E** $+2\mu_B B$

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2.1 C $m_l + 2m_s = 1 - 1 = 0$. This statement uses the uncoupled regime.

Compare the rejected options: $-2\mu_B B$; $-\mu_B B$; $+\mu_B B$; $+2\mu_B B$. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. An upper level shifts by $+2u$ and a lower level by $-u$. An allowed emitted photon shifts by: [1]

- **A** $-3u$
- **B** $-u$
- **C** 0
- **D** $+u$
- **E** $+3u$

2.3 For $L=1, S=1/2, J=1/2$, the Landé factor in the $g_s=2$ approximation is _____ (use a fraction). Show your relation and units. [2]

Exercises 2.2·2.3

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2.2 E Photon energy is $E_{\text{upper}} - E_{\text{lower}}$, so the change is $2u - (-u) = 3u$.

Compare the rejected options: $-3u$; $-u$; 0 ; $+u$. Use the stated model and conditions.

2.3 For $L=1, S=1/2, J=1/2$, the Landé factor in the $g_s=2$ approximation is _____ (use a fraction). Show your relation and units. [2]

2.3 $2/3$. $1 + [3/4 + 3/4 - 2] / (2 \cdot 3/4) = 2/3$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: A nondegenerate parity eigenstate's zero first-order Stark shift guarantees no electric-field response at any order. [2]

3.1 Two opposite-parity degenerate states have dipole perturbation matrix $\begin{pmatrix} 0 & dF \\ dF & 0 \end{pmatrix}$. Find energies and eigenstates to first order. Compare with two such states separated by a large unperturbed gap Δ . [5]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: A nondegenerate parity eigenstate's zero first-order Stark shift guarantees no electric-field response at any order. [2]

2.4 False. Off-diagonal dipole couplings can produce a quadratic shift; degeneracy changes the leading-order treatment.

3.1 Two opposite-parity degenerate states have dipole perturbation matrix $\begin{pmatrix} 0 & dF \\ dF & 0 \end{pmatrix}$. Find energies and eigenstates to first order. Compare with two such states separated by a large unperturbed gap Δ . [5]

3.1 Degenerate shifts are $\pm dF$, with eigenstates $(1, \pm 1)/\sqrt{2}$; splitting is $2|dF|$. With a large gap, each parity state's first-order diagonal shift is zero. Off-diagonal coupling instead gives ground shift approximately $-(dF)^2/\Delta$. Award shifts, states, splitting, vanishing nondegenerate first order and quadratic correction/condition.