

Exercise walk-through

# 4.1

## Thermodynamics and statistical mechanics

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GRE · GRE Subject Test · GRE Physics

# 4.1 Thermodynamics and statistical mechanics

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 11 marks**

**KEY WORDS** entropy • partition function

# Objective

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Apply the first and second laws with a sign convention; Relate ensembles, Boltzmann factors and entropy ; Analyse ideal gases, heat engines and quantum statistics. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

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State the first-law sign convention:  $\Delta U = Q - W$  when  $W$  is work done by the system.

For a reversible engine between reservoirs, efficiency is bounded by  $1 - T_c/T_h$  using absolute temperatures.

Boltzmann weights are  $\exp(-E/kT)$ . Probabilities require division by the partition function.

Distinguish Maxwell–Boltzmann, Bose–Einstein and Fermi–Dirac assumptions. Fermions cannot share an identical single-particle state.

■ **Worked method**

Use work  $W$  done by the gas as positive. The first law is

$$\Delta U = Q - W.$$

For reversible isothermal expansion of an ideal gas,  $U$  depends only on  $T$ , so

$$W = \int_{V_i}^{V_f} p dV = nRT \ln(V_f/V_i), \quad Q = W.$$

For  $n = 0.50$  mol,  $T = 300$  K and doubled volume,

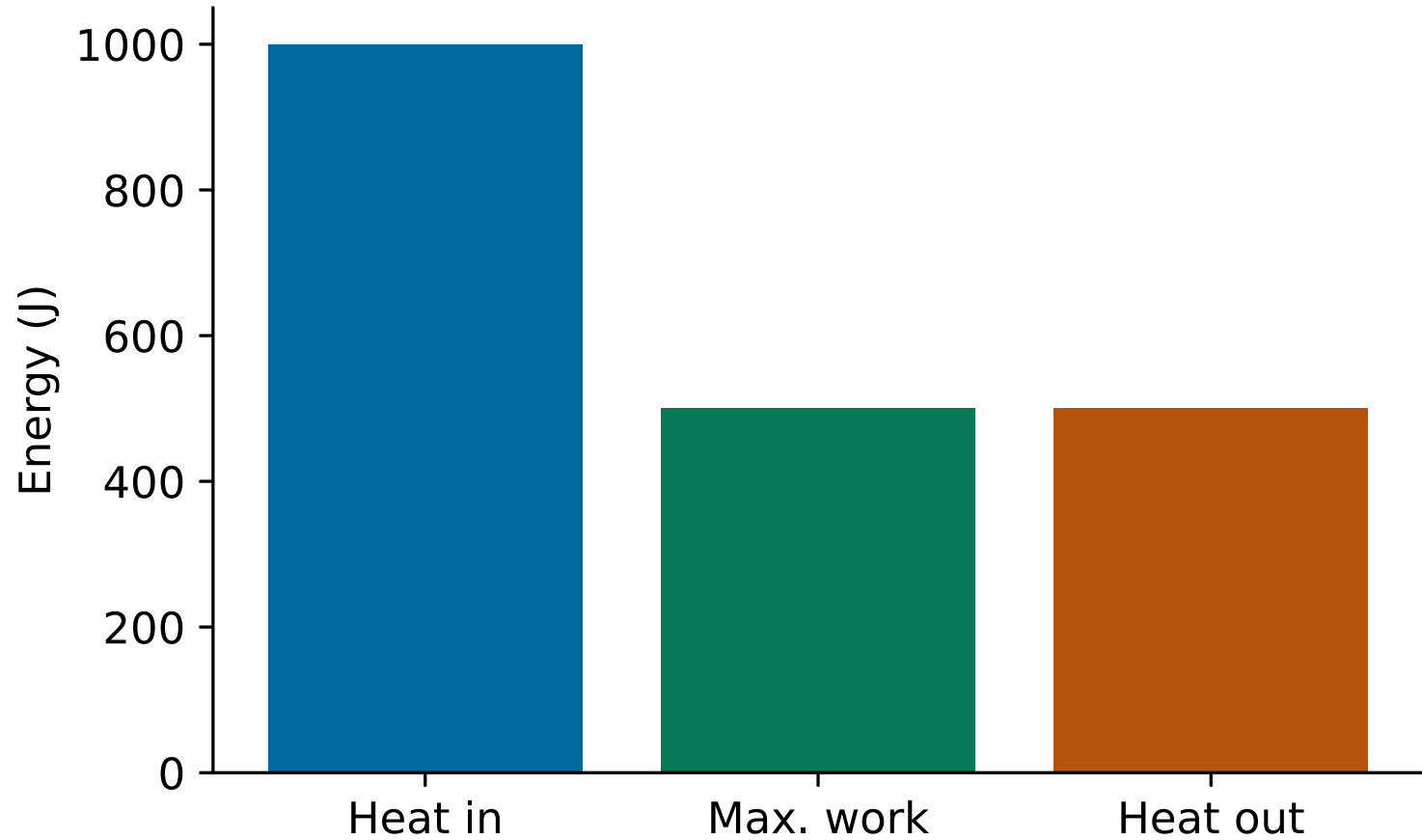
$$W = nRT \ln 2 = (0.50 \text{ mol})(8.314 \text{ J/(mol K)})(300 \text{ K}) \ln 2 = 864 \text{ J}.$$

$$\Delta S = Q_{rev}/T = (864 \text{ J})/(300 \text{ K}) = 2.88 \text{ J/K}.$$

The gas expands and receives heat. The reservoir loses the same entropy in this reversible limit.

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Carnot limit: hot 600 K, cold 300 K



## Exercises 2.1·2.2

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**2.1** Choose one answer. A Carnot engine between 400 K and 300 K has efficiency: [1]

- A 0.25
- B 0.75
- C 1.33

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**2.2** Choose one answer. A higher-energy state's Boltzmann weight at fixed  $T$  is: [1]

- A greater
- B smaller
- C always identical

## Exercises 2.1·2.2

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**2.1** Choose one answer. A Carnot engine between 400 K and 300 K has efficiency: [1]

- A 0.25
- B 0.75
- C 1.33

**2.1 A**  $1 - 300/400 = 0.25$ .

Compare the rejected options: 0.75; 1.33. Use the stated model and conditions.

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**2.2** Choose one answer. A higher-energy state's Boltzmann weight at fixed T is: [1]

- A greater
- B smaller
- C always identical

**2.2 B** The negative exponential decreases with energy.

Compare the rejected options: greater; always identical. Use the stated model and conditions.

## Exercises 2.3-3.1

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**2.3** A system receives 100 J heat and does 40 J work.  $\Delta U = \underline{\hspace{2cm}}$  J. Show your relation and units. [2]

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**2.4** Decide whether this claim is true. Justify it: An engine may convert all heat from one reservoir into work in a complete cycle with no other effect. [2]

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**3.1** An insulated rigid vessel contains ideal gas confined initially to half its volume. Remove the internal partition into vacuum. Compare  $Q$ ,  $W$ , temperature change and gas entropy change with reversible isothermal expansion to the same final volume. [5]

## Exercises 2.3·3.1

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**2.3** A system receives 100 J heat and does 40 J work.  $\Delta U = \underline{\hspace{2cm}}$  J. Show your relation and units. [2]

**2.3** 60.  $\Delta U = Q - W = 100 - 40$ .

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**2.4** Decide whether this claim is true. Justify it: An engine may convert all heat from one reservoir into work in a complete cycle with no other effect. [2]

**2.4** False. That violates the second law.

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**3.1** An insulated rigid vessel contains ideal gas confined initially to half its volume. Remove the internal partition into vacuum. Compare  $Q$ ,  $W$ , temperature change and gas entropy change with reversible isothermal expansion to the same final volume. [5]

**3.1** Free expansion has  $Q = 0$  by insulation and  $W = 0$  because external pressure is zero. Thus  $\Delta U = 0$  and ideal-gas temperature is unchanged. Entropy is a state function:  $\Delta S = nR \ln 2 > 0$ , evaluated on an imagined reversible isothermal path. The actual process is irreversible;  $Q_{\text{actual}}/T = 0$  is not its entropy change. Reversible expansion has  $Q = W = nRT \ln 2$  and the same gas entropy increase, with a matching reservoir entropy decrease. Award the four quantities and comparison.