

Exercise walk-through

32.1

Diffraction envelopes and missing interference orders

GRE · GRE Subject Test · GRE Physics

32.1 Diffraction envelopes and missing interference orders

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS diffraction envelope

- missing order

Objective

Derive and evaluate the far-field single-slit intensity using amplitude superposition; Separate double-slit interference spacing from finite-aperture envelope width; Identify missing orders and state the limits of far-field and small-angle formulas. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Uniform far-field slit amplitude is $\sin\beta/\beta$, $\beta=\pi a \sin\theta/\lambda$. Square it for intensity and take its centre limit as one. Zeros have a $\sin\theta=m\lambda$, $m\neq 0$.

For small angles, first-zero displacement is $L\lambda/a$ and central width $2L\lambda/a$. Use $\theta = a \sin(m\lambda/a)$, $y = L \tan\theta$ for larger angles; the far-field assumption remains necessary.

Two coherent equal finite slits give normalised $I = (\sin\beta/\beta)^2 \cos^2\alpha$, $\alpha = \pi d \sin\theta/\lambda$. Interference spacing $\approx \lambda L/d$; envelope width $\approx 2\lambda L/a$. Width and separation have different roles.

Missing orders satisfy $n = m d/a$. Integer $d/a = r$ cancels $\pm r, \pm 2r$, etc.; central-envelope nominal order centres satisfy $|n| < r$, giving $2r-1$. Exact product maxima can be slightly shifted.

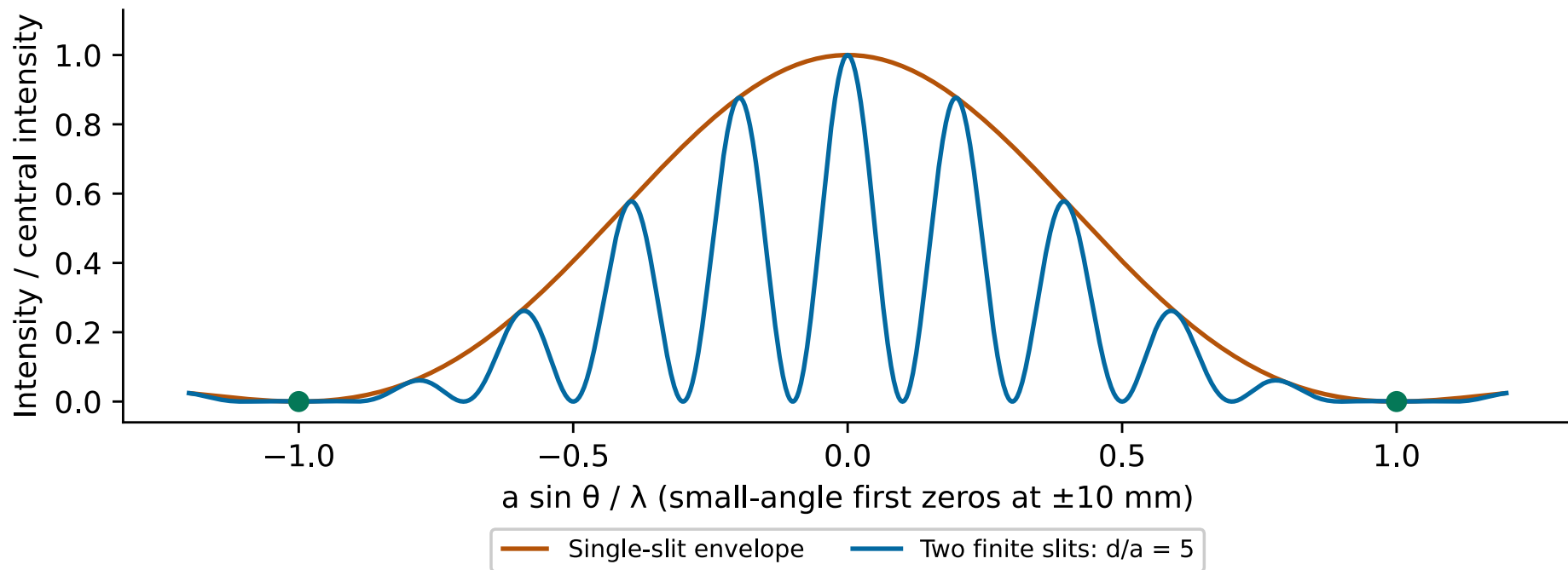
■ Worked method

For two finite slits, separate interference spacing from the single-slit envelope.

$$I/I_{centre} = (\sin \beta / \beta)^2 \cos^2 \alpha, \quad \beta = \pi a \sin \theta / \lambda, \quad \alpha = \pi d \sin \theta / \lambda.$$

If $d = 5a$, the first envelope zero $a \sin \theta = \lambda$ coincides with order $n = 5$. Nominal order centres in the central envelope are $n = -4$ through $+4$: nine in total. These are interference centres; the envelope can shift the exact product maxima slightly.

Orders ± 5 vanish; nine central order centres
 $\lambda = 500 \text{ nm}$, $a = 0.10 \text{ mm}$, $d = 0.50 \text{ mm}$, $L = 2 \text{ m}$



Exercise 2.1

2.1 Choose one answer. Two equal finite slits have $d=4a$. Which nominal interference orders are cancelled by the first single-slit minima? [1]

- **A** ± 1
- **B** ± 2
- **C** ± 3
- **D** ± 4
- **E** Only 0

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- **C** ± 3
- **D** ± 4
- **E** Only 0

2.1 D $d \sin\theta = n\lambda$ and $a \sin\theta = \pm\lambda$ coincide when $n = \pm d/a = \pm 4$.

Compare the rejected options: ± 1 ; ± 2 ; ± 3 ; Only 0. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. Double slit width a is doubled while d , λ and L stay fixed in the small-angle far field. What changes? [1]

- **A** Spacing and envelope both double
- **B** Fringe spacing halves; envelope fixed
- **C** Fringe spacing fixed; envelope width halves
- **D** Envelope doubles; fringe spacing fixed
- **E** All intensity zeros disappear

2.3 With $\lambda=600$ nm, $L=1$ m and $a=0.20$ mm, small-angle central single-slit width is _____ mm. Show your relation and units. [2]

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- **A** Spacing and envelope both double
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- **C** Fringe spacing fixed; envelope width halves
- **D** Envelope doubles; fringe spacing fixed
- **E** All intensity zeros disappear

2.2 C Spacing $\lambda L/d$ is unchanged; central envelope width $2\lambda L/a$ halves.

Compare the rejected options: Spacing and envelope both double; Fringe spacing halves; envelope fixed; Envelope doubles; fringe spacing fixed; All intensity zeros disappear. Use the stated model and conditions.

2.3 With $\lambda=600$ nm, $L=1$ m and $a=0.20$ mm, small-angle central single-slit width is _____ mm. Show your relation and units. [2]

2.3 6. $2L\lambda/a=0.006$ m=6 mm.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: The single-slit centre is dark because $\sin\beta/\beta$ becomes $0/0$ there. [2]

3.1 A double slit has $d = 3a/2$. Which positive interference orders are missing? How many nominal interference centres lie strictly between the first envelope zeros? Explain why rounding d/a to an integer is invalid. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: The single-slit centre is dark because $\sin\beta/\beta$ becomes $0/0$ there. [2]

2.4 False. The limit is one; the apparent $0/0$ is removable, and the centre is the intensity maximum.

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3.1 Coincidence requires $n = m(d/a) = 3m/2$ with both m, n nonzero integers. Thus m must be even and missing orders are $n = 3, 6, 9, \dots$. Central centres obey $|n| < 3/2$, so $n = -1, 0, 1$: three. Fractional d/a describes actual geometry; rounding changes which zeros coincide. Award integer condition, missing sequence, count and explanation.