

Exercise walk-through

24.1

Scattering, degeneracy and Pauli operators

GRE · GRE Subject Test · GRE Physics

24.1 Scattering, degeneracy and Pauli operators

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS reflection coefficient • degeneracy

Objective

Match travelling waves and fluxes across finite potential changes; Count degenerate isotropic-oscillator states; Evaluate Pauli products and distinguish anticommutation from equality. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For constant potential with $E > V$, $k = \sqrt{2m(E - V)}/\hbar$ and solutions travel as $\exp(\pm ikx)$. At finite steps of equal mass, ψ and ψ' are continuous. No incoming wave from the right leaves only a right-travelling transmitted wave. A finite well returning to its external potential has equal incident/transmitted external k .

Step matching gives $r=(k_1-k_2)/(k_1+k_2)$, $t=2k_1/(k_1+k_2)$. Reflection $R=|r|^2$; transmission $T=(k_2/k_1)|t|^2$ uses probability current. $R+T=1$ for a lossless step. Downward steps can reflect; raw squared transmitted amplitude is not always its current fraction.

A spin-zero 3D isotropic oscillator has energy $(N+3/2)\hbar\omega$, $N=n_x+n_y+n_z \geq 0$. Ordered nonnegative triples number $(N+1)(N+2)/2$. For $N=2$, permutations of $(2,0,0)$ and $(1,1,0)$ give six states. Extra spin/identical-particle constraints require separate counting.

Pauli matrices $\sigma_x=[[0,1],[1,0]]$, $\sigma_y=[[0,-i],[i,0]]$, $\sigma_z=[[1,0],[0,-1]]$ each square to I . Cyclic products give $\sigma_x\sigma_y=i\sigma_z$; reversed order changes sign. Thus distinct matrices anticommute, and $\sigma_x\sigma_z=-i\sigma_y$. Physical spin is $S_i=\hbar\sigma_i/2$.

■ Worked method

At a finite potential step, match wavefunction and derivative for the same particle mass.

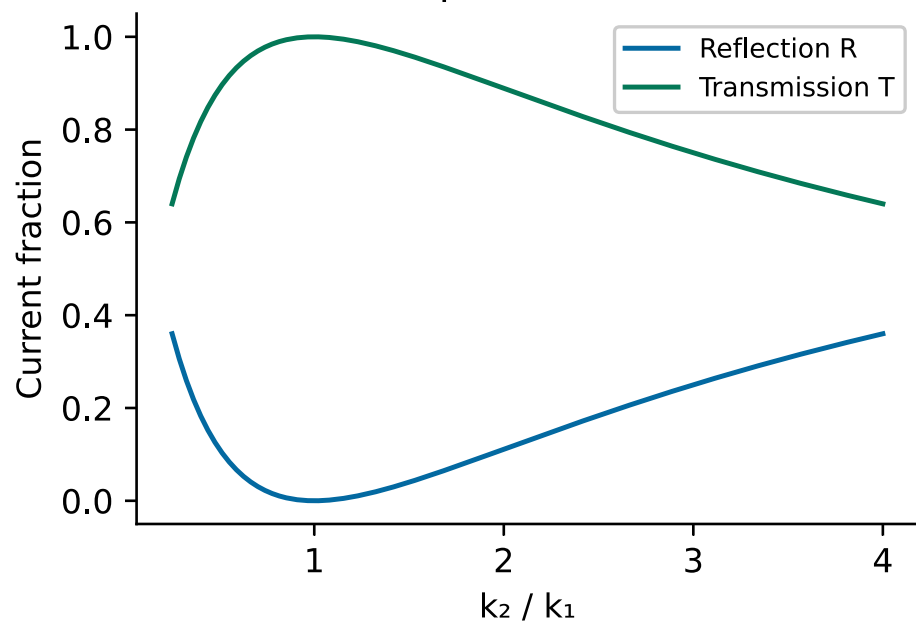
$$r = \frac{k_1 - k_2}{k_1 + k_2}, \quad t = \frac{2k_1}{k_1 + k_2}.$$

Transmission probability uses current, not amplitude alone. For $k_2 = 3k_1$,
 $r = -1/2$, $t = 1/2$.

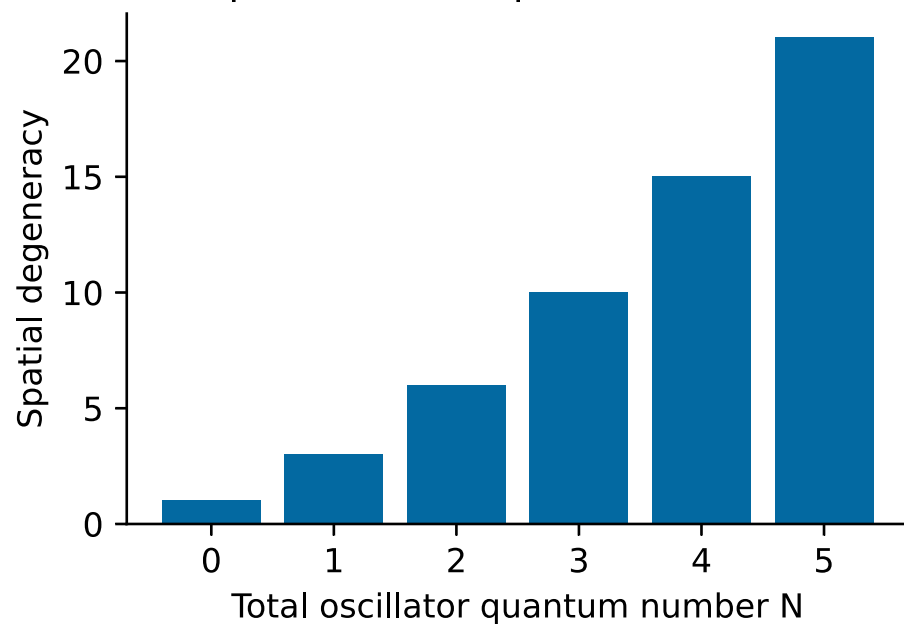
$$R = |r|^2 = 1/4, \quad T = (k_2/k_1)|t|^2 = 3/4, \quad R + T = 1.$$

Reflection occurs even for a downward step. Its phase sign disappears when taking probability.

Above both potentials: $R + T = 1$



Spin-zero isotropic 3D oscillator



Exercise 2.1

2.1 Choose one answer. For a lossless potential step with $k_2=2k_1$, reflection probability is: [1]

- **A** 0
- **B** $1/9$
- **C** $1/3$
- **D** $4/9$
- **E** 1

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- **E** 1

2.1 B $r=(1-2)/(1+2)=-1/3$; $R=1/9$.

Compare the rejected options: 0; $1/3$; $4/9$; 1. Use the stated model and conditions.

Exercises 2.2-2.3

2.2 Choose one answer. A spin-zero three-dimensional isotropic oscillator at $E=(5/2)\hbar\omega$ has degeneracy: [1]

- A 1
- B 2
- C 3
- D 4
- E 6

2.3 For a spin-zero isotropic oscillator with $N=4$, spatial degeneracy is _____. Show your relation and units. [2]

Exercises 2.2·2.3

2.2 Choose one answer. A spin-zero three-dimensional isotropic oscillator at $E=(5/2)\hbar\omega$ has degeneracy: [1]

- A 1
- B 2
- C 3
- D 4
- E 6

2.2 C $N=1$ gives triples $(1,0,0),(0,1,0),(0,0,1)$.

Compare the rejected options: 1; 2; 4; 6. Use the stated model and conditions.

2.3 For a spin-zero isotropic oscillator with $N=4$, spatial degeneracy is _____. Show your relation and units. [2]

2.3 15. $(N+1)(N+2)/2=5 \cdot 6/2=15$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: Distinct Pauli matrices commute with each other. [2]

3.1 A spin-half particle has state $(\sqrt{3}, i)^T/2$ in the z-spin basis. Check normalisation and find probabilities for S_z and S_x outcomes. [5]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: Distinct Pauli matrices commute with each other. [2]

2.4 False. Their products change sign on reversing order; they anticommute.

3.1 A spin-half particle has state $(\sqrt{3}, i)^T/2$ in the z-spin basis. Check normalisation and find probabilities for S_z and S_x outcomes. [5]

3.1 Norm is $(3+1)/4 = 1$. S_z gives $+\hbar/2$ with probability $3/4$ and $-\hbar/2$ with probability $1/4$. The x eigenstates are $(1, \pm 1)^T/\sqrt{2}$. Their amplitudes are $(\sqrt{3} \pm i)/(2\sqrt{2})$, each with squared modulus $1/2$. Award norm, z results/probabilities, x eigenbasis, projections and x probabilities. Complex conjugation is required in the inner product; the imaginary relative phase matters.