

Exercise walk-through

# 22.1

## Relativistic lifetime, energy and Doppler shift

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GRE · GRE Subject Test · GRE Physics

# 22.1 Relativistic lifetime, energy and Doppler shift

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 10 marks**

**KEY WORDS** proper time • rest energy

# Objective

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Relate proper lifetime to laboratory time and distance; Use invariant energy–momentum and relativistic kinetic energy; Infer longitudinal recession speed from wavelength ratio. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

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For relative speed  $v$ , define  $\beta=v/c$  and  $\gamma=1/\sqrt{1-\beta^2}$ . Proper time is measured along the particle's worldline by a clock at rest with it. If its proper mean lifetime is  $\tau_0$ , its laboratory mean lifetime is  $\gamma\tau_0$  and mean travel distance is  $v\gamma\tau_0$  at constant speed. The decay is statistical: mean lifetime is not a guaranteed decay time for each particle. At  $\beta=0.8$ ,  $\gamma=5/3$ . Compute spacetime events consistently in one frame; proper time and

coordinate time are different quantities.

Total energy is  $E=\gamma mc^2$ , momentum  $p=\gamma mv$  and invariant  $E^2-p^2c^2=m^2c^4$ . When  $E$  and  $pc$  are expressed in the same energy units, find  $mc^2=\sqrt{E^2-(pc)^2}$ , not  $E-pc$ . The nonnegative square root is required for positive rest mass. For  $E=13$  GeV and  $pc=12$  GeV, rest energy is 5 GeV and mass is 5 GeV/c<sup>2</sup>. A massless particle has  $E=pc$  but need not have zero energy or momentum. These relations concern isolated-particle four-momentum, not classical  $mv$  at relativistic speed.

Work accelerating a particle from rest equals kinetic energy  $K=E-mc^2=(\gamma-1)mc^2$ . At  $\beta=0.6$ ,  $\gamma=1.25$  and  $K=0.25mc^2$ , while the classical  $\frac{1}{2}mv^2$  gives  $0.18mc^2$ . Classical kinetic energy is the low-speed expansion and becomes inaccurate near  $c$ . Finite acceleration work increases  $\gamma$  rather than allowing a massive particle to reach or exceed  $c$ . Keep total energy, rest energy and kinetic energy distinct when interpreting answer units.

For purely longitudinal relative recession in special relativity, wavelength ratio  $r=\lambda_{\text{observer}}/\lambda_{\text{source}}$

Rearranging gives  $\beta=(r^2-1)/(r^2+1)$ . A ratio  $r=2$  gives  $\beta=3/5$ , not  $c$  times  $r-1$  from a low-speed approximation. Blueshift uses  $r<1$  and a negative recession parameter under this convention. The formula assumes the shift is entirely kinematic; cosmological expansion or gravitational redshift requires a different model. State the question's stipulated model before interpreting a spectral ratio.

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## ■ Worked method

Total energy and momentum determine invariant rest energy.

$$E^2 - (pc)^2 = (mc^2)^2, \quad mc^2 = \sqrt{E^2 - (pc)^2}.$$

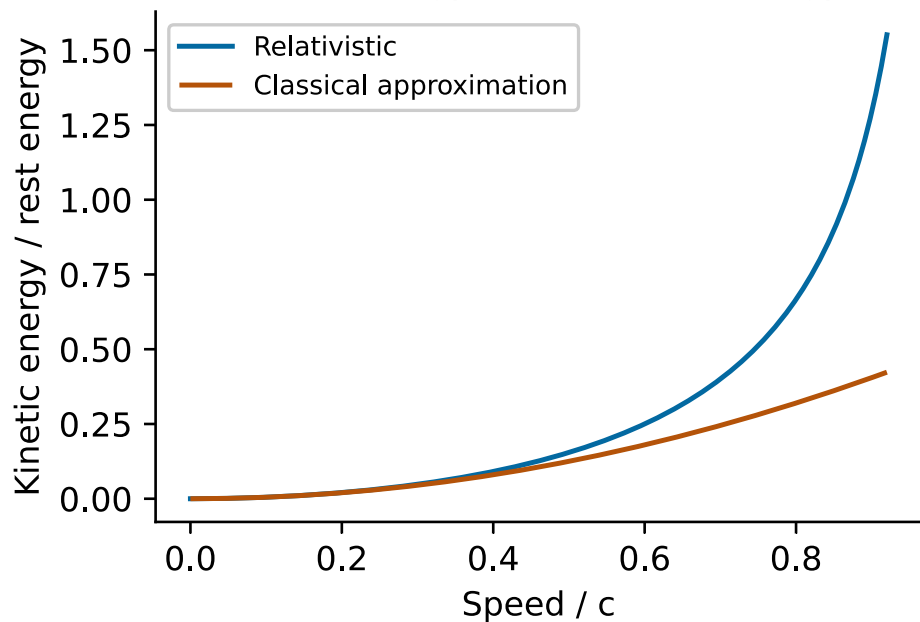
For  $E = 17 \text{ GeV}$  and  $pc = 15 \text{ GeV}$ ,

$$mc^2 = \sqrt{(17 \text{ GeV})^2 - (15 \text{ GeV})^2} = 8 \text{ GeV}.$$

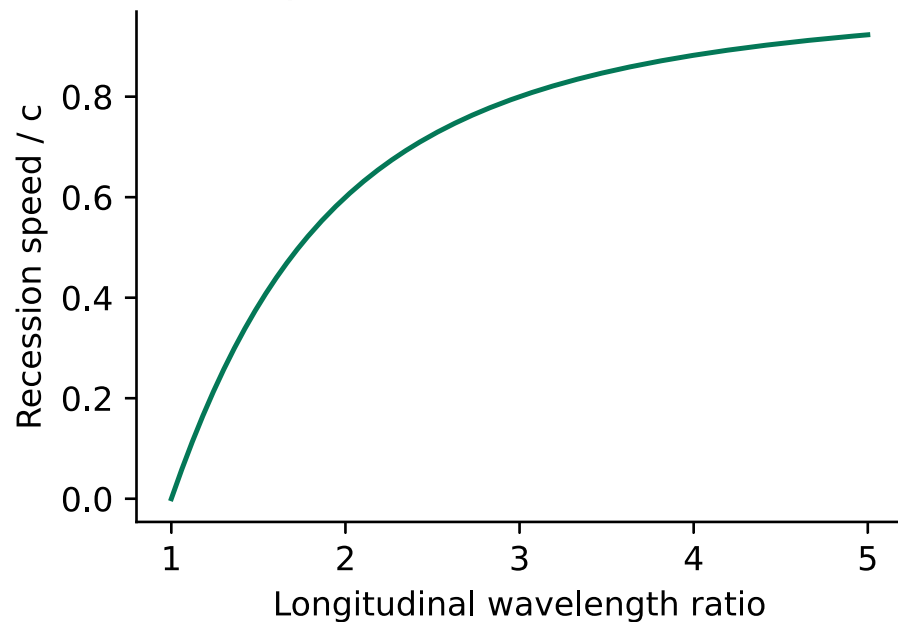
$$K = E - mc^2 = 17 \text{ GeV} - 8 \text{ GeV} = 9 \text{ GeV}.$$

Rest mass is 8 GeV/c squared. Do not subtract momentum directly from energy.

Total energy minus rest energy



Stipulated kinematic redshift



## Exercise 2.1

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**2.1** Choose one answer. A particle at  $0.8c$  has proper mean lifetime  $6 \mu\text{s}$ . Its laboratory mean lifetime is: [1]

- **A**  $3.6 \mu\text{s}$
- **B**  $4.8 \mu\text{s}$
- **C**  $6 \mu\text{s}$
- **D**  $10 \mu\text{s}$
- **E**  $30 \mu\text{s}$

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- E  $30 \mu\text{s}$

**2.1 D**  $\gamma=5/3$ , so  $\gamma\tau_0=10 \mu\text{s}$ .

Compare the rejected options:  $3.6 \mu\text{s}$ ;  $4.8 \mu\text{s}$ ;  $6 \mu\text{s}$ ;  $30 \mu\text{s}$ . Use the stated model and conditions.

## Exercises 2.2·2.4

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**2.2** Choose one answer. For  $E=10$  GeV and  $pc=6$  GeV, rest energy is: [1]

- **A** 4 GeV
- **B** 6 GeV
- **C** 8 GeV
- **D** 10 GeV
- **E** 16 GeV

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**2.3** At speed  $0.6c$ ,  $K/(mc^2)=$ \_\_\_\_\_ (decimal). Show your relation and units. [2]

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**2.4** Decide whether this claim is true. Justify it: A wavelength redshift ratio of 2 implies superluminal recession under the stipulated longitudinal special-relativity model. [2]

## Exercises 2.2-2.4

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- A 4 GeV
- B 6 GeV
- C 8 GeV
- D 10 GeV
- E 16 GeV

**2.2** C  $\sqrt{100-36}=8$  GeV.

Compare the rejected options: 4 GeV; 6 GeV; 10 GeV; 16 GeV. Use the stated model and conditions.

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**2.3** At speed  $0.6c$ ,  $K/(mc^2)=$ \_\_\_\_\_ (decimal). Show your relation and units. [2]

**2.3** 0.25.  $\gamma=1.25$ , so  $\gamma-1=0.25$ .

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**2.4** Decide whether this claim is true. Justify it: A wavelength redshift ratio of 2 implies superluminal recession under the stipulated longitudinal special-relativity model. [2]

**2.4** False.  $\beta=(4-1)/(4+1)=0.6$ .

## Exercise 3.1

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**3.1** Equal-energy photons of energy  $E$  collide with directions separated by angle  $\theta$ . Derive the invariant mass of the pair. Check parallel and head-on limits. [4]

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**3.1** Total energy is  $2E$ ; total momentum magnitude is  $2(E/c)|\cos(\theta/2)|$ .  $M^2c^4 = 4E^2 -$