

Exercise walk-through

21.1

Counting statistics, uncertainty and dimensional models

GRE · GRE Subject Test · GRE Physics

21.1 Counting statistics, uncertainty and dimensional models

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS standard uncertainty

- Poisson approximation

Objective

Use binomial and Poisson count means and fluctuations; Propagate small uncertainties with stated correlation assumptions; Solve dimensional exponent constraints and check a model's units. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For N independent detection trials with probability p , binomial mean is Np and variance $Np(1-p)$. Standard deviation is its square root. For large N and small p , Poisson mean/variance are $\lambda=Np$. Stable efficiency does not guarantee a fixed count; correlations or dead time can invalidate independence.

Linearised independent uncertainties combine as $\sigma_y^2 \approx \sum (\partial y / \partial x_i)^2 \sigma_i^2$. Correlations need covariance terms. A power $y = x^a$ has fractional uncertainty about $|a| \sigma_x / |x|$. For $K = \frac{1}{2}mv^2$, independent mass/speed contributions are combined as $\sqrt{(\sigma_m/m)^2 + (2\sigma_v/v)^2}$. Use small standard errors, not unstated bounds.

Random scatter measures precision; calibration bias affects accuracy. Independent mean uncertainty falls as $1/\sqrt{n}$, but common bias does not. Physical spread and uncertainty of its mean differ. State whether percentages are standard errors, confidence intervals or worst-case bounds.

For $G^a \hbar^b c^d$ to be a length, dimensional powers give $-a+b=0$, $3a+2b+d=1$ and $-2a-b-d=0$. Hence $a=b=1/2$, $d=-3/2$ and length $\sqrt{G\hbar/c^3}$. Unit consistency cannot determine a dimensionless coefficient or validate physical assumptions.

■ Worked method

For independent small standard uncertainties in $K = mv^2/2$, differentiate first.

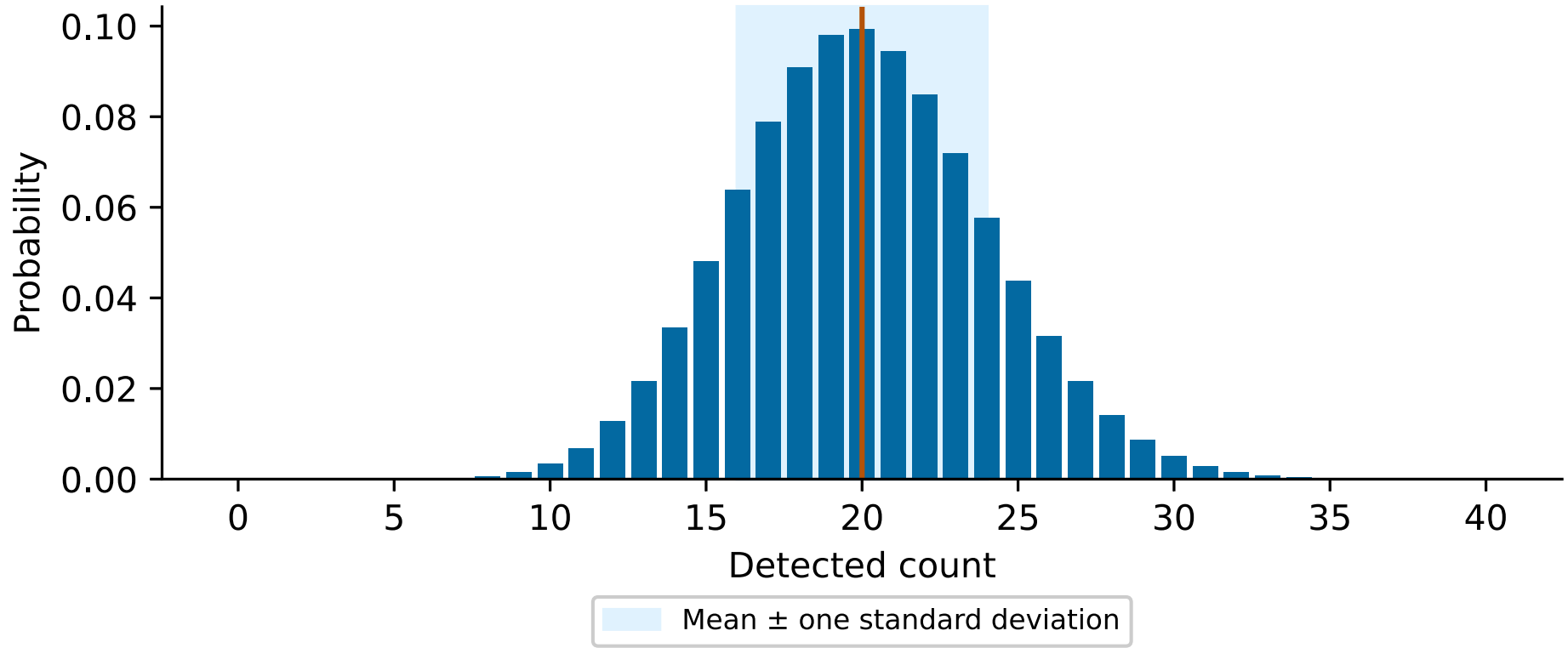
$$\left(\frac{\sigma_K}{K}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(2\frac{\sigma_v}{v}\right)^2.$$

With relative uncertainties 3% in mass and 2% in speed,

$$\sigma_K/K = \sqrt{0.03^2 + (2 \times 0.02)^2} = 0.05.$$

A worst-case bound would use a sum, not this quadrature. Correlations require covariance terms.

Binomial $N = 100$, $p = 0.20$: mean 20, deviation 4



Exercise 2.1

2.1 Choose one answer. A binomial detector has $N=100$ and $p=0.20$. Its mean and standard deviation are: [1]

- **A** 20 and 20
- **B** 20 and 4
- **C** 20 and 16
- **D** 5 and 2
- **E** Exactly 20 with no deviation

Exercise 2.1

2.1 Choose one answer. A binomial detector has $N=100$ and $p=0.20$. Its mean and standard deviation are: [1]

- **A** 20 and 20
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- **C** 20 and 16
- **D** 5 and 2
- **E** Exactly 20 with no deviation

2.1 B Mean 20; variance $100 \cdot 0.2 \cdot 0.8=16$, so standard deviation 4.

Compare the rejected options: 20 and 20; 20 and 16; 5 and 2; Exactly 20 with no deviation. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. With negligible mass uncertainty and speed uncertainty 3%, small kinetic-energy fractional uncertainty is: [1]

- A 1.5%
- B 3%
- C 6%
- D 9%
- E 12%

2.3 A Poisson count has mean 25. Its standard deviation is _____. Show your relation and units. [2]

Exercises 2.2-2.3

2.2 Choose one answer. With negligible mass uncertainty and speed uncertainty 3%, small kinetic-energy fractional uncertainty is: [1]

- A 1.5%
- B 3%
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- E 12%

2.2 C K depends on v^2 , so the fractional uncertainty doubles.

Compare the rejected options: 1.5%; 3%; 9%; 12%. Use the stated model and conditions.

2.3 A Poisson count has mean 25. Its standard deviation is _____. Show your relation and units. [2]

2.3 5. Poisson variance equals mean, and standard deviation is its square root.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: A dimensionally consistent formula necessarily has the correct numerical coefficient and physical assumptions. [2]

3.1 A sample interval contains 900 counts; an independent equal-duration background interval contains 100. Estimate net counts and their standard uncertainty
. If both intervals are doubled at stable rates, what happens to relative uncertainty?[4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: A dimensionally consistent formula necessarily has the correct numerical coefficient and physical assumptions. [2]

2.4 False. Units do not determine dimensionless factors or validate the physical model.

3.1 A sample interval contains 900 counts; an independent equal-duration background interval contains 100. Estimate net counts and their standard uncertainty
. If both intervals are doubled at stable rates, what happens to relative uncertainty?[4]

3.1 Net count is 800. Independent Poisson variances add despite subtraction: $\sigma = \sqrt{900 + 100} = \sqrt{1000} = 31.62$ counts, relative 0.03953. Doubled intervals give net 1600 and variance 2000, so relative uncertainty is divided by $\sqrt{2}$. Award net, variance addition, uncertainty and time scaling. This assumes stable rates and no dead-time correlations.