

Exercise walk-through

19.1

Gas processes, entropy and reversible cycles

GRE · GRE Subject Test · GRE Physics

19.1 Gas processes, entropy and reversible cycles

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS entropy production • coefficient of performance

Objective

Apply first-law work and heat signs to gas processes; Compare reversible entropy balances and P–V cycle areas; Distinguish engine efficiency from refrigerator coefficient of performance . Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Use $\Delta U = Q - W$, with heat entering and work done by the gas positive. Quasistatic work is $\int P \, dV$. Ideal-gas U depends on temperature: constant volume gives $Q = \Delta U$; isothermal gives $Q = W$; adiabatic gives $Q = 0$. A complete cycle has $\Delta U = 0$, hence net $Q = \text{net } W$.

Reversible isothermal paths have PV constant; with constant heat capacities, adiabatic paths have PV^γ constant and fall more steeply. Isobaric paths are horizontal. Clockwise

P–V cycles give positive work: rectangle area $\Delta P \Delta V$, triangle area half that. Convert litres to m^3 .

Reversible heat transfer gives $dS = \delta Q_{\text{rev}}/T$. Total system-plus-environment entropy change is zero, but system entropy can change: isothermal expansion gives $nR \ln(V_2/V_1)$, balanced by reservoir entropy loss. Irreversibility produces nonnegative total entropy; reversible does not mean isothermal or zero work.

Carnot engine efficiency is $1 - T_C/T_H$. Refrigerator $\text{COP} = Q_C/W = T_C/(T_H - T_C)$; heat-pump $\text{COP} = T_H/(T_H - T_C)$, using kelvin. On a reversible T–S rectangle, isothermal heat is $T \Delta S$ and work magnitude is the area. Refrigerator COP can exceed one because it moves heat while consuming work.

■ Worked method

A reversible refrigerator transfers entropy ΔS from T_c to T_h .

$$Q_c = T_c \Delta S = (270 \text{ K})(3.0 \text{ J/K}) = 810 \text{ J}.$$

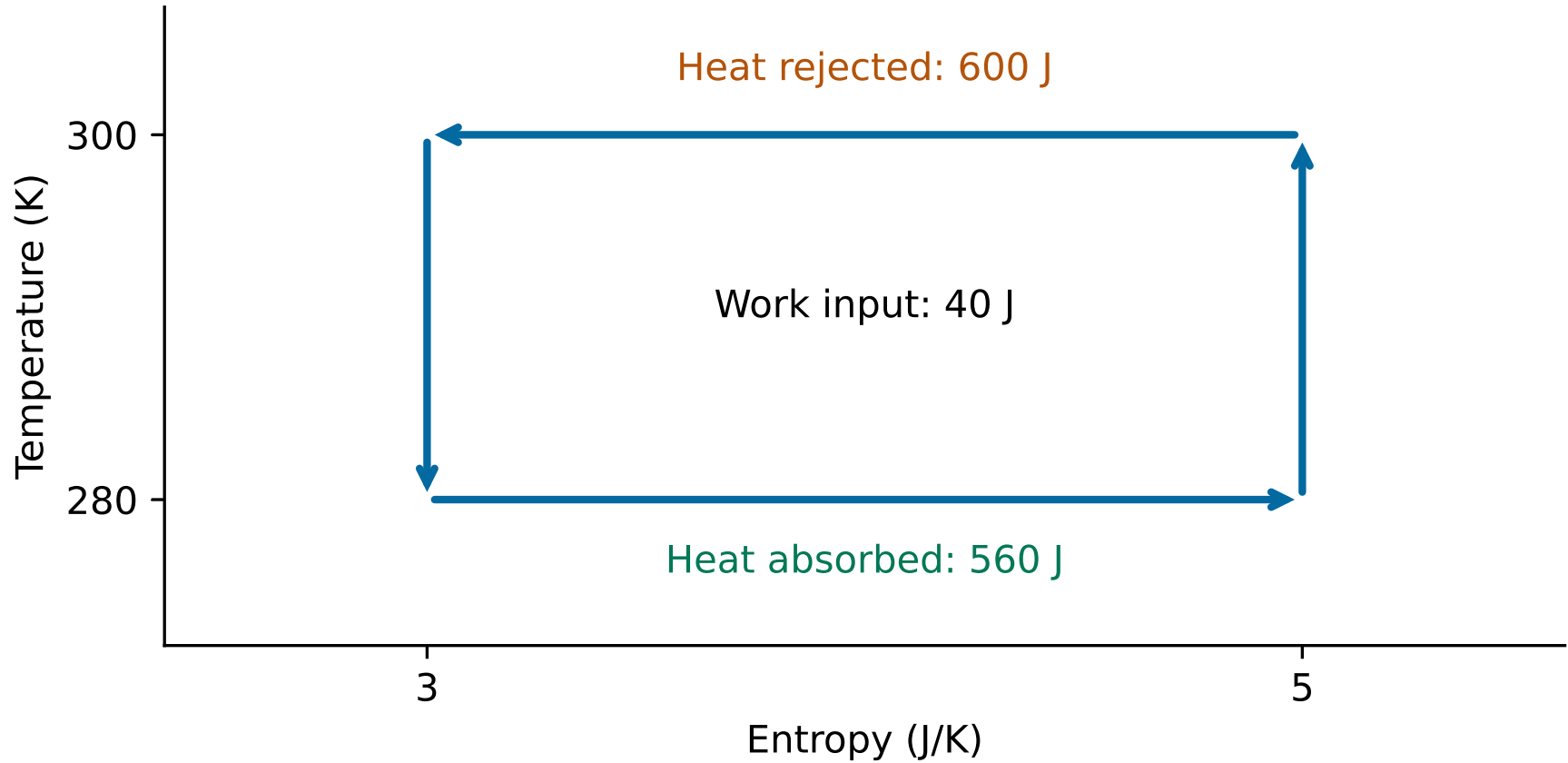
$$Q_h = T_h \Delta S = (300 \text{ K})(3.0 \text{ J/K}) = 900 \text{ J}.$$

$$W_{in} = Q_h - Q_c = 900 \text{ J} - 810 \text{ J} = 90 \text{ J}.$$

$$\text{COP} = Q_c / W_{in} = (810 \text{ J}) / (90 \text{ J}) = 9.$$

COP above one is possible because heat is moved, rather than all produced from work.

Reversible refrigerator: $\Delta S = 2 \text{ J/K}$; COP = 14



Exercise 2.1

2.1 In a reversible process of a system and its environment, which must hold? [1]

- A. The system entropy is constant
- B. Temperature is constant
- C. Total entropy change is zero
- D. Work is zero
- E. Internal energy is constant

Exercise 2.1

2.1 In a reversible process of a system and its environment, which must hold? [1]

- A. The system entropy is constant
- B. Temperature is constant
- C. Total entropy change is zero
- D. Work is zero
- E. Internal energy is constant

2.1 Answer: C. Reversibility produces no total entropy; individual system quantities may change.

Compare the rejected options: The system entropy is constant; Temperature is constant; Work is zero; Internal energy is constant. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 A reversible refrigerator has $T_C=270$ K and $T_H=300$ K. Its COP is: [1]

- A. 0.1
- B. 0.9
- C. 1
- D. 9
- E. 10

2.3 A clockwise rectangular P–V cycle has $\Delta P=4000$ Pa and $\Delta V=0.002$ m³. Net work is _____ J. Show your relation and units. [2]

Exercises 2.2·2.3

2.2 A reversible refrigerator has $T_C=270$ K and $T_H=300$ K. Its COP is: [1]

A. 0.1

B. 0.9

C. 1

D. 9

E. 10

2.2 Answer: D. $Q_C/W=270/(300-270)=9$.

Compare the rejected options: 0.1; 0.9; 1; 10. Use the stated model and conditions.

2.3 A clockwise rectangular P–V cycle has $\Delta P=4000$ Pa and $\Delta V=0.002$ m³. Net work is _____ J. Show your relation and units. [2]

2.3 8. Positive clockwise work is $\Delta P\Delta V=8$ J.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: For any complete gas cycle, net entering heat equals net work done by the gas. [2]

3.1 A refrigerator removes 600 J from 280 K and rejects 650 J to 300 K per cycle. Check energy balance and total entropy change. Is the proposed cycle possible? Find the minimum work needed for the same cooling between these reservoirs. [5]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: For any complete gas cycle, net entering heat equals net work done by the gas. [2]

2.4 True. $\Delta U=0$ because internal energy is a state function and the initial state returns.

3.1 A refrigerator removes 600 J from 280 K and rejects 650 J to 300 K per cycle. Check energy balance and total entropy change. Is the proposed cycle possible? Find the minimum work needed for the same cooling between these reservoirs. [5]

3.1 Energy requires 50 J work input. Reservoir entropy change is $650/300 - 600/280 = +0.02381 \text{ J/K} > 0$, so the complete cycle does not violate the second law. Reversible minimum work is $Q_c(T_h/T_c - 1) = 600(300/280 - 1) = 42.86 \text{ J}$. The proposed cycle is possible thermodynamically and uses more than minimum work. Award work, correct entropy arithmetic, sign/possibility, reversible bound and comparison.