

Exercise walk-through

18.1

Fourier symmetry and wave superposition

GRE · GRE Subject Test · GRE Physics

18.1 Fourier symmetry and wave superposition

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS orthogonality • Fourier coefficient

Objective

Use orthogonality to identify Fourier coefficients; Exploit even and odd parity without discarding the constant term; Relate component amplitudes to interference and physical waveforms. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For a real 2π -periodic function, $f = a_0/2 + \sum [a_n \cos(nx) + b_n \sin(nx)]$. Coefficients are integrals of f times the relevant basis function over $[-\pi, \pi]$, divided by π . Orthogonality separates components. The mean is $a_0/2$, not a_0 .

Even f about the chosen origin has all $b_n = 0$ because $f \sin(nx)$ is odd. Odd f has zero

mean and all $a_n=0$. Shifting the origin can mix sine/cosine coefficients without changing harmonic frequencies. An even nonnegative triangular wave can have cosine terms and a nonzero mean.

For periodic $|x|$ on $[-\pi, \pi]$, integration by parts gives mean $\pi/2$ and $a_n=2[(-1)^{n-1}]/(\pi n^2)$. Thus only odd cosine coefficients survive, at $-4/(\pi n^2)$. Its continuous cusp gives $1/n^2$ decay; discontinuous waveforms need a different convergence analysis.

Equal coherent monochromatic fields at phase difference φ give intensity $2I_0(1+\cos\varphi)$: $4I_0$ in phase and zero at π . Incoherent averaging gives $2I_0$. Fourier sums add fields with their phases; a waveform's instantaneous value is not a sum of individual intensities.

■ Worked method

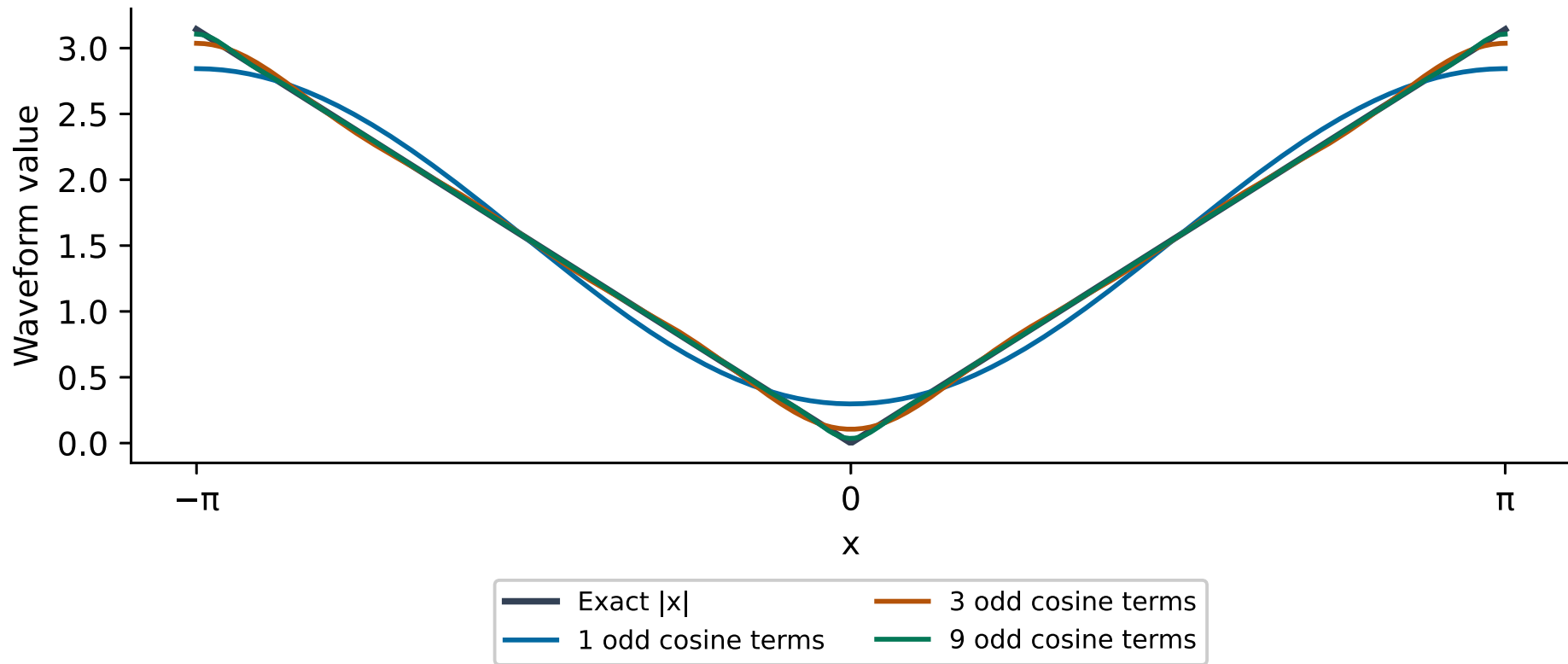
For a real periodic function, use $f = a_0/2 + \sum (a_n \cos nx + b_n \sin nx)$. Even symmetry makes all sine coefficients zero because the integrand is odd. For $f(x) = |x|$ on

$[-\pi, \pi],$

$$a_n = \frac{2}{\pi} \int_0^\pi x \cos(nx) dx = \frac{2[(-1)^n - 1]}{\pi n^2}.$$

Integration by parts supplies the second expression. The mean is $a_0/2 = \pi/2$; only odd cosine harmonics survive. A cusp does not justify dropping the constant term.

Even triangular wave: no sine coefficients



Exercise 2.1

2.1 Choose one answer. A real even 2π -periodic waveform about $x=0$ necessarily has: [1]

- **A** All $a_n=0$
- **B** All $b_n=0$
- **C** Zero mean
- **D** Only its fundamental
- **E** Equal a_n and b_n

Exercise 2.1

2.1 Choose one answer. A real even 2π -periodic waveform about $x=0$ necessarily has: [1]

- **A** All $a_n=0$
- **B** All $b_n=0$
- **C** Zero mean
- **D** Only its fundamental
- **E** Equal a_n and b_n

2.1 B Even times sine is odd, so each symmetric sine-coefficient integral is zero.

Compare the rejected options: All $a_n=0$; Zero mean; Only its fundamental; Equal a_n and b_n . Use the stated model and conditions.

Exercises 2.2·2.4

2.2 Choose one answer. For the periodic extension of $|x|$, a_2 is:

[1]

- **A** $-4/\pi$
- **B** $-1/\pi$
- **C** 0
- **D** $\pi/2$
- **E** 2

2.3 If $a_0=6$, the waveform mean is _____. Show your relation and units.

[2]

2.4 Decide whether this claim is true. Justify it: Shifting the origin can mix sine and cosine coefficients without changing the harmonic frequencies.

[2]

Exercises 2.2-2.4

2.2 Choose one answer. For the periodic extension of $|x|$, a_2 is: [1]

- A $-4/\pi$
- B $-1/\pi$
- C 0
- D $\pi/2$
- E 2

2.2 C $a_n = 2[(-1)^{n-1}]/(\pi n^2)$; even n gives zero.

Compare the rejected options: $-4/\pi$; $-1/\pi$; $\pi/2$; 2. Use the stated model and conditions.

2.3 If $a_0=6$, the waveform mean is _____. Show your relation and units. [2]

2.3 3. The constant term is $a_0/2$.

2.4 Decide whether this claim is true. Justify it: Shifting the origin can mix sine and cosine coefficients without changing the harmonic frequencies. [2]

2.4 True. The angle-addition identities mix coefficients at the same frequency.

Exercise 3.1

3.1 Shift this waveform by $\pi/2$: $g(x) = f(x - \pi/2)$. Write its first nonconstant harmonic and its mean. Explain why its sine coefficient need not vanish, although the unshifted waveform is even. [4]

Exercise 3.1

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3.1 Substitute the shift in $f \approx \pi/2 - (4/\pi) \cos x$: $g \approx \pi/2 - (4/\pi) \sin x$. Its mean remains $\pi/2$. Evenness depends on the chosen origin; g is not even about zero. Frequencies stay unchanged. Award substitution, harmonic/sign, mean and origin/frequency interpretation.