

Exercise walk-through

10.1

Collisions, work and oscillator energy

GRE · GRE Subject Test · GRE Physics

10.1 Collisions, work and oscillator energy

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS external impulse

- turning point

Objective

Compute vector momentum and energy loss in sticking collisions; Compare fixed-force springs and use work–energy conditions; Derive small-oscillation periods and oscillator energies. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

With negligible external impulse, conserve momentum in each direction. For sticking masses, $\mathbf{v} = (m_1\mathbf{v}_1 + m_2\mathbf{v}_2) / (m_1 + m_2)$. Add vectors before taking magnitudes. Compare $\Sigma \frac{1}{2}m|\mathbf{v}|^2$ initially with $\frac{1}{2}(m_1 + m_2)|\mathbf{v}|^2$ finally; the loss becomes internal energy/deformation. Sticking does not mean final rest.

Work–energy gives $\Delta K = \int \mathbf{F} \cdot d\mathbf{r}$, or Fs for constant parallel force. Mechanical-energy

conservation requires no unaccounted nonconservative work. Spring energy is $\frac{1}{2}kx^2$. At fixed force, $x=F/k$ and $U=F^2/(2k)$; at fixed extension U increases with k . These comparisons have opposite answers.

Near stable equilibrium, $U \approx U_0 + \frac{1}{2}k_{\text{eff}}(x-x_0)^2$ with $k_{\text{eff}}=U''(x_0)>0$. Then $\omega=\sqrt{k_{\text{eff}}/m}$, $T=2\pi/\omega$. A small-angle pendulum uses $\sin\theta \approx \theta$ in $\theta\ddot{+}(g/L)\sin\theta=0$, giving $T=2\pi\sqrt{L/g}$. Large amplitudes need a correction; ideal pendulum mass cancels.

Undamped oscillator energy is $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. At equilibrium it is all kinetic; at turning points it is all potential. Use $\omega=2\pi f$ and $A=v_{\text{max}}/\omega$. The measurement position determines which energy expression is simplest; convert joules to millijoules only after calculation.

■ **Worked method**

Two carts stick with negligible external impulse. Add momentum vectors $\vec{p}_1, \vec{p}_2$, then

divide by total mass. Known: $m_1 = 2 \text{ kg}$, $\mathbf{v}_1 = (3, 0) \text{ m/s}$; $m_2 = 1 \text{ kg}$, $\mathbf{v}_2 = (0, 3) \text{ m/s}$.

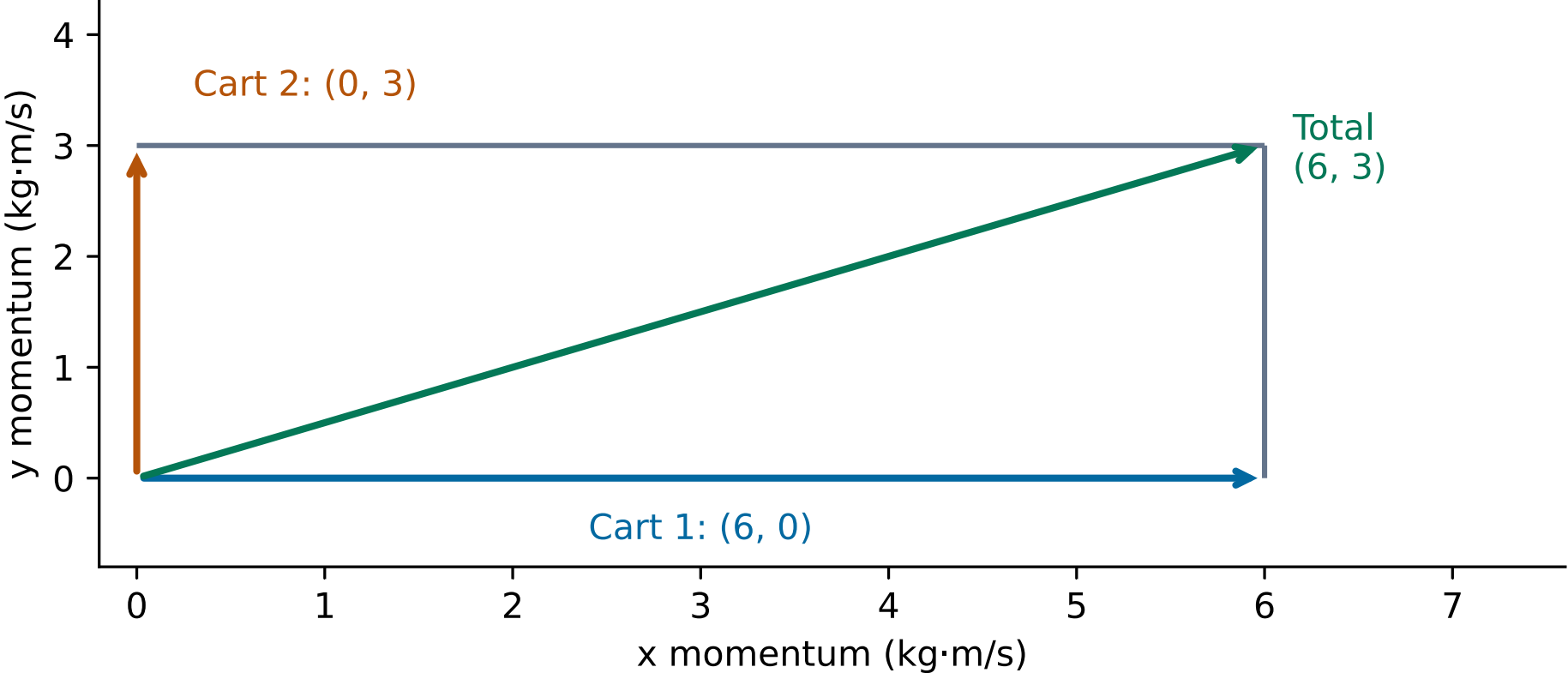
$$\mathbf{v}_f = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = \frac{(2 \text{ kg})(3, 0) \text{ m/s} + (1 \text{ kg})(0, 3) \text{ m/s}}{3 \text{ kg}} = (2, 1) \text{ m/s}.$$

$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(2 \text{ kg})(3 \text{ m/s})^2 + \frac{1}{2}(1 \text{ kg})(3 \text{ m/s})^2 = 13.5 \text{ J}.$$

$$K_f = \frac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 = \frac{1}{2}(3 \text{ kg})(5 \text{ m}^2/\text{s}^2) = 7.5 \text{ J}.$$

The 6 J difference becomes internal energy. Kinetic energy is not conserved in sticking.

Add momentum components before dividing by mass



Exercise 2.1

2.1 Choose one answer. For the two carts in the worked example, the common velocity is: [1]

- **A** (3,3) m/s
- **B** (2,1) m/s
- **C** (1,2) m/s
- **D** (2,2) m/s
- **E** (0,0) m/s

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2.1 B Divide the vector momentum (6,3) by total mass 3, component by component.

Compare the rejected options: (3,3) m/s; (1,2) m/s; (2,2) m/s; (0,0) m/s. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. Two springs with $k_1=3k_2$ hold the same force. The ratio U_1/U_2 is: [1]

- A 3
- B 9
- C 1
- D $1/3$
- E $1/9$

2.3 A small-angle pendulum length is multiplied by 9 at fixed g . Its period is multiplied by _____. Show your relation and units. [2]

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- D $1/3$
- E $1/9$

2.2 D At fixed force $U=F^2/(2k)$, hence the inverse stiffness ratio.

Compare the rejected options: 3; 9; 1; $1/9$. Use the stated model and conditions.

2.3 A small-angle pendulum length is multiplied by 9 at fixed g . Its period is multiplied by _____. Show your relation and units. [2]

2.3 3. T scales as $\sqrt{L}$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: At equilibrium an undamped harmonic oscillator can have nonzero kinetic energy and zero spring potential energy. [2]

3.1 Two equal masses approach with speed u along directions separated by angle θ and stick. Derive final speed and the fraction of initial kinetic energy lost. Check parallel, perpendicular and opposite incoming directions. [5]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: At equilibrium an undamped harmonic oscillator can have nonzero kinetic energy and zero spring potential energy. [2]

2.4 True. Displacement is zero, so spring energy is zero; speed is maximal.

3.1 Two equal masses approach with speed u along directions separated by angle θ and stick. Derive final speed and the fraction of initial kinetic energy lost. Check parallel, perpendicular and opposite incoming directions. [5]

3.1 The sum of equal velocity vectors has magnitude $2u|\cos(\theta/2)|$. Dividing momentum by total mass gives $v_f = u|\cos(\theta/2)|$. $K_i = mu^2$ and $K_f = mv_f^2$, so lost fraction is $\sin^2(\theta/2)$. The three limits are 0, 1/2 and 1. Award vector sum, speed, energy relation, lost fraction and limits. The formula uses $0 \leq \theta \leq \pi$ between directions.