

Measurement loading, amplifier gain and detector response

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A voltmeter changes the circuit it measures; the reading is a divider output, not the open-circuit voltage. Calculate measurement loading from finite meter and source impedances Determine simple amplifier gain and first-order filter response Interpret detector efficiency, resolution, dead time and calibration bias

Model the measurement

Reasoning

Model the measured circuit as a Thevenin source V_{th} with resistance R_{th} and the instrument as input resistance R_m . The reading is $V_{th} \cdot R_m / (R_m + R_{th})$, with the same sign as V_{th} and magnitude no greater than $|V_{th}|$ for positive resistances. A 10 V source with $R_{th}=100 \text{ k}\Omega$ read by a $1 \text{ M}\Omega$ meter gives $10 \times (1/1.1) = 9.09 \text{ V}$, a 9% low bias. A $10\times$ oscilloscope probe raises the effective input resistance and divides the signal by ten; quote both effects.

Set gain and cutoff

Reasoning

An ideal non-inverting op-amp stage has gain $1+R_2/R_1$; the inverting stage has $-R_2/R_1$, with the sign carried explicitly. A first-order RC low-pass has cutoff $f_c=1/(2\pi RC)$; its amplitude ratio is $1/\sqrt{1+(f/f_c)^2}$ and its phase is $-\arctan(f/f_c)$. At $f=f_c$ the amplitude is $1/\sqrt{2}$ (the -3 dB point), not one half. For $R=1\text{ k}\Omega$ and $C=100\text{ nF}$, $f_c\approx 1.59\text{ kHz}$.

Rate the detector

Reasoning

Detector efficiency ε is detected events divided by incident events; energy resolution is quoted as FWHM/E, such as 13.2 keV on a 662 keV line, about 2%. A non-paralyzable detector with dead time τ records $m=n/(1+n\tau)$ from true rate n ; inverting gives $n=m/(1-m\tau)$, which fails as $m\tau \rightarrow 1$. State which dead-time model you assume; paralyzable behaviour differs.

Calibrate with its limits

Reasoning

A linear calibration against known standards can correct offset and scale error; it does not remove noise or guarantee correction of nonlinearity. A calibration curve maps indicated to true values; interpolation between points assumes local smoothness. Report resolution and efficiency separately: sharp peaks with poor efficiency still miss events, and high efficiency with broad resolution still mixes nearby lines.

Worked reasoning 1

Reasoning

A voltmeter of finite input resistance forms a divider with the source resistance.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$V_m = V_{th} \frac{R_m}{R_m + R_{th}} = (12 \text{ V}) \frac{1.0 \text{ M}\Omega}{1.0 \text{ M}\Omega + 0.20 \text{ M}\Omega} = 10.0 \text{ V}.$$

Worked reasoning 3

Reasoning

For a nonparalysable detector with observed rate m and dead time τ ,

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

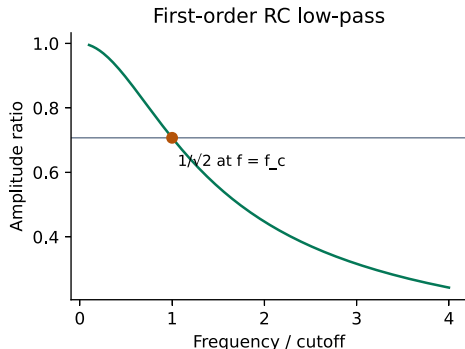
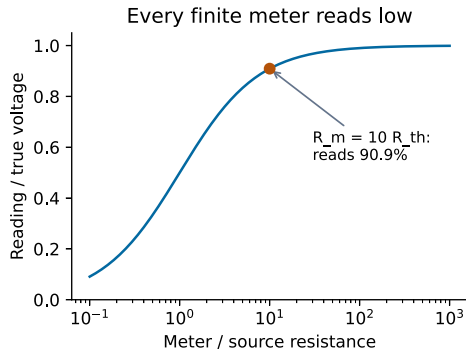
$$n = m/(1 - m\tau), \quad m\tau < 1.$$

Worked reasoning 5

Reasoning

Efficiency, dead time and spectral resolution are different corrections; one cannot replace the others.

Read the original diagram



- Original GRE teaching diagram: Measurement loading, amplifier gain and detector response
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

The same source is measured first with a 1 megohm meter, giving 6 V, then with a 3 megohm meter, giving 9 V. Find its Thevenin voltage and resistance. Assume stable linear behaviour. **[4]**

Independent transfer · reasoning

Reasoning

Let resistance r be in megohms. The first reading gives $V_{th} = 6(1 + r)$. The second gives $V_{th} = 9(3 + r)/3 = 9 + 3r$. Equating yields $r = 1$, hence $R_{th} = 1$ megohm and $V_{th} = 12$ V. Both readings are below the unloaded voltage. Award two divider equations, resistance and voltage/check.

Check assumptions and limits

Reasoning

Treating the $1/\sqrt{2}$ cutoff as one half, ignoring that a finite meter reads low through the divider, or applying the non-paralyzable correction beyond its range.

Keep the technical distinctions

Reasoning

dead time : The minimum interval after one recorded event during which the detector cannot record another. input impedance : The effective load a measuring instrument presents to the circuit under test.

Check yourself

- Define dead time and give its condition of use.
- Define input impedance and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.