

Simultaneity, length measurement and velocity transformation

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

Two observers can disagree about which of two events happened first, and both can be right. Transform event separations and identify proper length versus simultaneous endpoint readings Apply relativistic velocity addition and check a four-vector invariant Classify the interval sign to order events and rule out causal contact

Transform separations

Reasoning

Use the Lorentz transformation with declared frame motion: if frame S' moves at $+v$ along x relative to S , then $x' = \gamma(x - vt)$ and $t' = \gamma(t - vx/c^2)$ with $\gamma = 1/\sqrt{1 - v^2/c^2}$. Apply it to event separations Δx and Δt , not only to single coordinates. Two events simultaneous in S ($\Delta t = 0$) at different places have $\Delta t' = -\gamma v \Delta x / c^2$: the event further in the $+x$ direction happens earlier in S' . Simultaneity is frame-dependent, not a bookkeeping error.

Measure length in one frame

Reasoning

A moving rod length must be read from endpoint positions at the same lab time ($\Delta t=0$). Then $\Delta x' = \gamma \Delta x$ gives $L = L_0 / \gamma$ with L_0 the rest-frame (proper) length. Reading the endpoints simultaneously in the rod frame instead does not produce the lab length; the two reading conditions answer different questions. State the frame and the reading condition before quoting any length.

Classify the interval

Reasoning

The interval $s^2 = c^2 \Delta t^2 - \Delta x^2$ (mostly-minus convention, declared once) is invariant. $s^2 > 0$ is timelike: a frame exists with the events co-located, and $\Delta \tau = s/c$ is the proper time between them. $s^2 < 0$ is spacelike: a frame exists with the events simultaneous, and $\sqrt{-s^2}$ is the proper distance. $s^2 = 0$ is lightlike. Timelike order is the same in every frame; spacelike order can swap, and no signal can join the events because $|\Delta x / \Delta t| > c$.

Add velocities and check invariants

Reasoning

Collinear velocities add as $u = (u' + v) / (1 + u'v/c^2)$; the denominator keeps $u < c$ for any $u', v < c$, and $u' = c$ gives $u = c$ exactly. Energy and momentum form a four-vector with invariant $E^2 - p^2 c^2 = m^2 c^4$: compute it in any convenient frame. Check each result in the nonrelativistic limit, where the addition rule reduces to $u = u' + v$ and the invariant reduces to the rest energy.

Worked reasoning 1

Reasoning

Two lab events have $\Delta x = 900 \text{ m}$ and $\Delta t = 2 \text{ microseconds}$. Their spacetime interval is spacelike because $c \Delta t = 600 \text{ m}$ is less than Δx . A frame making them simultaneous satisfies

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

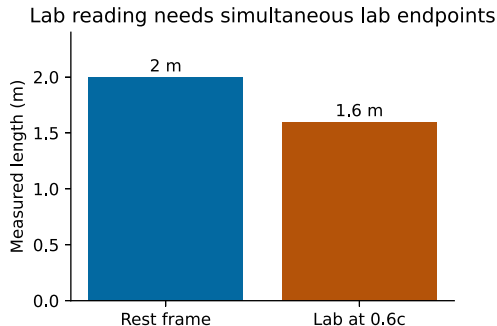
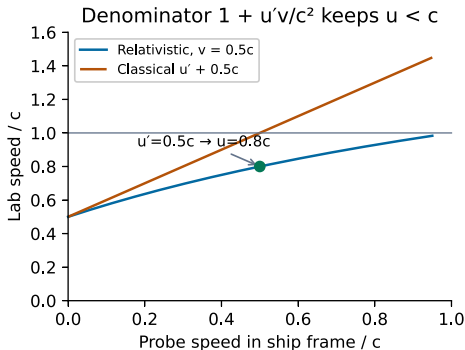
$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0, \quad v = c^2 \Delta t / \Delta x = 2c/3.$$

Worked reasoning 3

Reasoning

Their separation in that frame is $\sqrt{\Delta x^2 - c^2 \Delta t^2} = 670.8 \text{ m}$. No frame can put these spacelike events at the same position.

Read the original diagram



- Original GRE teaching diagram: Simultaneity, length measurement and velocity transformation
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Two events are separated by 300 m and 2 microseconds in the lab. Find the velocity of a frame in which they occur at one position, and their time separation there. Can a frame reverse their order? **[4]**

Independent transfer · reasoning

Reasoning

Set $\Delta x' = \gamma(\Delta x - v\Delta t) = 0$, so $v = \Delta x / \Delta t = 0.5c$. The proper time is $\sqrt{\Delta t^2 - (\Delta x/c)^2} = \sqrt{3}$ microseconds. They are timelike separated, and every time-oriented inertial frame retains their order. Award velocity condition, speed, proper time and causal classification/order.

Check assumptions and limits

Reasoning

Quoting $L=L_0/\gamma$ without the simultaneous-endpoint condition, or adding velocities as $u=u'+v$ at relativistic speeds. A simultaneity difference is physical, not a mistake.

Keep the technical distinctions

Reasoning

spacetime interval : The invariant combination $c^2\Delta t^2 - \Delta x^2$ under the declared sign convention, whose sign classifies an event separation. proper length

: The endpoint separation measured in the object's rest frame; any other frame must read both endpoints simultaneously in its own time.

Check yourself

- Define spacetime interval and give its condition of use.
- Define proper length and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.