

Polarisation, magnetisation and material fields

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

Putting a dielectric into a capacitor changes the field differently depending on whether its battery remains connected. Relate free and bound charge to D , E and polarisation
Compare fixed-charge and fixed-voltage dielectric changes Use magnetic constitutive response and free-current boundary conditions

Separate free and bound charge

Reasoning

Electric polarisation $\mathbf{P}$ is electric dipole moment per unit volume, measured in C/m^2 . Define $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ so that $\nabla \cdot \mathbf{D} = \rho_{\text{free}}$; the total charge, including bound charge, still appears in $\nabla \cdot \mathbf{E} = \rho_{\text{total}}/\epsilon_0$. Bound volume charge is $\rho_{\text{b}} = -\nabla \cdot \mathbf{P}$ and bound surface charge is $\sigma_{\text{b}} = \mathbf{P} \cdot \mathbf{n}$, where $\mathbf{n}$ points outward from the material. Uniform $\mathbf{P}$ therefore gives no bound charge in the bulk but can give surface charges of opposite sign. A dielectric is not an ideal metal: polarisation need not make its internal electric field zero.

Apply the material relation

Reasoning

For a linear isotropic dielectric, $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$ and $\mathbf{D} = \epsilon \mathbf{E}$ with $\epsilon = \epsilon_0(1 + \chi_e) = \epsilon_0 \epsilon_r$. These simple scalar relations assume the response is linear and ignore anisotropy, strong dispersion and nonlinear effects. In a fully filled, large parallel-plate capacitor with negligible edge effects, $\mathbf{D}$ normal to the plates equals the free surface-charge density. If σ_{free} stays fixed, increasing ϵ reduces $E = \sigma_{\text{free}} / \epsilon$. The bound charges oppose the applied field; their magnitude is not automatically equal to the free plate charge.

Hold charge or voltage fixed

Reasoning

For plate area A and spacing d , $C = \epsilon A/d$. Disconnecting the battery fixes free charge Q : $V = Q/C$ falls and stored energy $U = Q^2/(2C)$ falls when a dielectric increases C .

Leaving an ideal voltage source connected fixes V : $Q = CV$ and stored energy $U = CV^2/2$ both rise. The source supplies energy and moving the dielectric can involve mechanical work; compare the stated electrical energy quantity rather than assuming the capacitor alone is an isolated system. At a material interface $\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_{\text{free}}$, whereas static tangential $\mathbf{E}$ is continuous. Normal $\mathbf{E}$ generally changes when permittivity changes.

Distinguish magnetic fields

Reasoning

Magnetisation M is magnetic dipole moment per unit volume, measured in A/m. In SI, $B = \mu_0(H + M)$; for a linear isotropic response $M = \chi_m H$ and $B = \mu_0(1 + \chi_m)H$. B and H have different units and roles. A long uniform solenoid has $H \approx nI$ when end and demagnetising effects are negligible; material response then changes B . Free surface current sets $n \times (H_2 - H_1) = K_{\text{free}}$, and normal B remains continuous. Ferromagnetic hysteresis and saturation cannot be represented by one constant χ_m over every field.

Worked reasoning 1

Reasoning

A capacitor is disconnected before inserting a linear dielectric with relative permittivity 4. Free charge stays fixed; capacitance becomes four times its original value.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$V' = Q/(4C) = V/4, \quad U' = Q^2/(2 \times 4C) = U/4.$$

Worked reasoning 3

Reasoning

If an ideal battery stays connected instead, V is fixed:

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

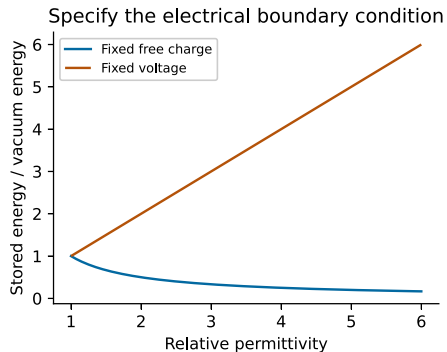
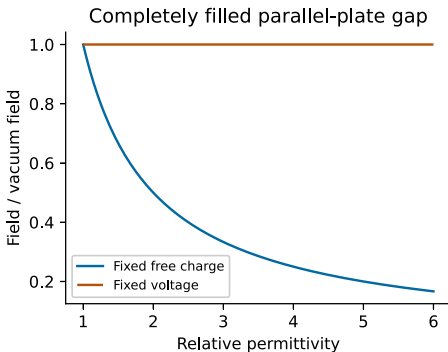
$$Q' = 4CV = 4Q, \quad U' = \frac{1}{2}(4C)V^2 = 4U.$$

Worked reasoning 5

Reasoning

The source exchanges energy. Do not compare isolated charge and fixed voltage as the same experiment.

Read the original diagram



- Original GRE teaching diagram: Polarisation, magnetisation and material fields
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A slab fills half the thickness of a parallel-plate capacitor and has permittivity $3\epsilon_0$; the remaining thickness is vacuum. Find C relative to the empty capacitor, and the two normal fields at fixed free charge Q . Ignore edge effects. **[5]**

Independent transfer · reasoning

Reasoning

Layers are in series. With no free interface charge, $D = Q/A$ is common.

$E_{vac} = Q/(\epsilon_0 A)$ and $E_d = Q/(3\epsilon_0 A)$. $V = (d/2)(E_{vac} + E_d) = 2Qd/(3\epsilon_0 A)$, hence $C = 3\epsilon_0 A/(2d) = 3C_0/2$. Award common D , each field, voltage sum and capacitance. Normal E is discontinuous; D is continuous.

Check assumptions and limits

Reasoning

Use free charge in the D equation and total charge in the E equation. Do not confuse fixed Q with fixed V , or identify B numerically with H .

Keep the technical distinctions

Reasoning

electric polarisation : Electric dipole moment per unit volume, P , contributing bound charge. magnetisation : Magnetic dipole moment per unit volume, M , entering $B = \mu_0(H + M)$.

Check yourself

- Define electric polarisation and give its condition of use.
- Define magnetisation and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.