

Coupled acceleration and relative motion

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A load released from an aircraft keeps the aircraft's horizontal velocity. Relative to the aircraft, its initial motion can therefore be purely downward. Solve coupled-body equations using one acceleration constraint Transform velocities between inertial frames Check force and energy limits for constrained motion

Choose the system and model

Reasoning

For a mass m_1 on a frictionless horizontal table connected over an ideal massless pulley to a hanging mass m_2 , choose table motion and downward hanging motion as positive. A taut inextensible string gives one acceleration magnitude. Write $T = m_1 a$ and $m_2 g - T = m_2 a$. Adding eliminates the internal tension, so $a = m_2 g / (m_1 + m_2)$ and $T = m_1 m_2 g / (m_1 + m_2)$. Thus $T < m_2 g$ when both masses are positive. Setting tension equal to hanging weight silently assumes zero acceleration.

Use the governing relation

Reasoning

The common tension assumption requires an ideal massless string and pulley with negligible friction and inertia. A pulley with rotational inertia can have different tensions, with $(T_2 - T_1)R = I\alpha$ and $a = R\alpha$ if the string does not slip. Draw each body separately: a force internal to the combined system remains an external force on one chosen body. Constraint forces can cancel from a system equation while still being needed for individual motion.

Apply the conditions

Reasoning

For two inertial frames with constant relative velocity V , Galilean velocity transformation is $v_{\text{relative}} = v_{\text{ground}} - V$. A payload released by a level aircraft initially has the aircraft's horizontal speed. With negligible air resistance, horizontal ground velocity remains constant and vertical velocity becomes $-gt$ if upward is positive. Relative to that aircraft, horizontal velocity is zero and downward speed is gt . Position is a different question: vertical displacement is $-\frac{1}{2}gt^2$. Do not confuse ground speed with relative speed.

Check the conclusion

Reasoning

Check limiting cases after solving. As m_1 approaches zero, the ideal coupled acceleration approaches g and tension approaches zero; as m_1 grows very large, acceleration approaches zero and tension approaches m_2g from below. An energy derivation gives $m_2g s = \frac{1}{2}(m_1 + m_2)v^2$ for release from rest and an ideal pulley, agreeing with $v^2 = 2as$. This agreement checks the equations; it does not permit energy conservation if friction or other unaccounted work is present.

Worked reasoning 1

Reasoning

A table mass m_1 and hanging mass m_2 share an acceleration through a taut ideal string. Choose rightward table motion and downward hanging motion positive.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$T = m_1 a, \quad m_2 g - T = m_2 a.$$

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{(2 \text{ kg})(10 \text{ m/s}^2)}{3 \text{ kg} + 2 \text{ kg}} = 4 \text{ m/s}^2.$$

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

$$T = m_1 a = (3 \text{ kg})(4 \text{ m/s}^2) = 12 \text{ N}.$$

Worked reasoning 7

Reasoning

T is less than the hanging weight 20 N. A nonzero net downward force is needed for acceleration.

Read the original diagram

Subtract aircraft velocity; components in m/s

Velocity after 3 s	Horizontal	Vertical
Ground	80	-30
Aircraft	80	0
Payload relative to aircraft	0	-30

- Original GRE teaching diagram: Coupled acceleration and relative motion
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Replace the ideal pulley by a solid disk of mass M and radius R , rotating without axle friction. The string does not slip. Derive the acceleration for table mass m_1 and hanging mass m_2 . Find the two tensions and check the limit M approaching zero.

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Independent transfer · reasoning

Reasoning

$T_1 = m_1 a$; $m_2 g - T_2 = m_2 a$; $(T_2 - T_1)R = I\alpha$. With $I = MR^2/2$ and $\alpha = a/R$,
 $T_2 - T_1 = Ma/2$. Thus $a = m_2 g / (m_1 + m_2 + M/2)$, $T_1 = m_1 m_2 g / (m_1 + m_2 + M/2)$
 and $T_2 = m_2(g - a)$. At $M = 0$ tensions become equal and the ideal result returns.
 Award body equations, torque/constraint, acceleration, tensions and limit.

Check assumptions and limits

Reasoning

A massless ideal pulley equates tension, not tension and weight. Subtract frame velocities component by component before taking a speed magnitude.

Keep the technical distinctions

Reasoning

constraint : A condition linking allowed positions or motions of connected bodies.
relative velocity : Velocity of one object measured in another moving frame.

Check yourself

- Define constraint and give its condition of use.
- Define relative velocity and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.