5.1 Quantum mechanics and atomic physics

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS normalisation • operator

Objective

Use wavefunctions, operators and measurement probabilities; Solve standard bound-state models and tunnelling reasoning; Apply angular momentum, spin and atomic spectral principles. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Normalise so $\int|\psi|^2 dx=1$. In an orthonormal expansion $\psi=\Sigma c_j\varphi_j$ the norm squared is $\Sigma|c_j|^2$; each energy probability is $|c_j|^2$.

Hermitian observables have real eigenvalues. The expectation $<A>=\int\psi^*\ A\psi\ dx$ is a repeated-preparation average. Apply operator identities to eigenstates: $A^4=I$ gives only real eigenvalues $+1$ and -1.

For an infinite well $0<x<L$, zero endpoint values give sine modes with $E_n=n^2\pi^2\hbar^2/(2mL^2)$, $n\geq1$. Doubling L quarters the energies. Finite barriers permit exponential tails.

Commuting observables share an eigenbasis. $T=p^2/(2m)$ commutes with p; a variable V(x) generally breaks $[H,p]=0$. $[J_x,J_y]=i\hbar J_z$; J^2 commutes with each component. Photon energies equal level differences, subject to selection rules.

■ Worked method

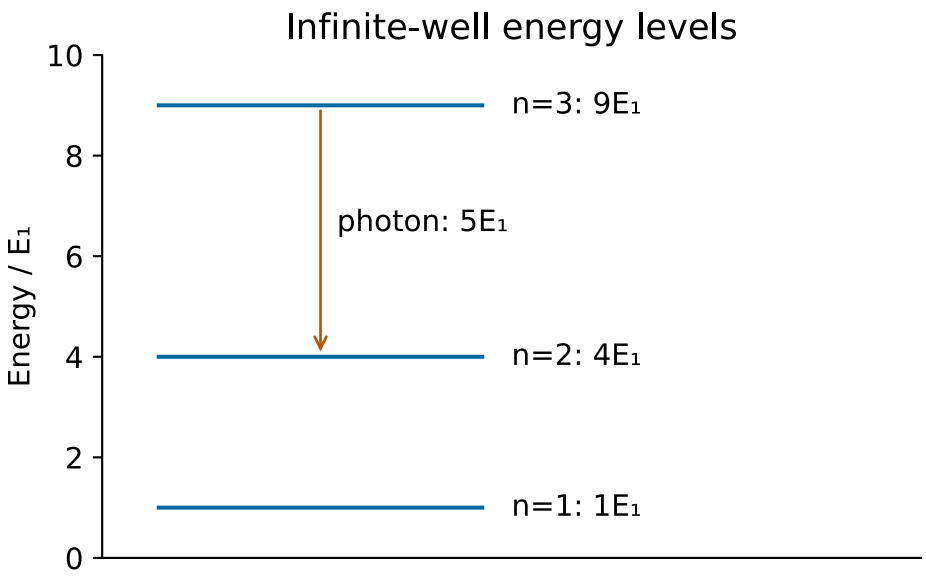
An infinite well extends from 0 to L. Vanishing wavefunction at both ends gives

$$\psi_n = \sqrt{2/L}\sin(n\pi x/L), \qquad E_n = n^2\pi^2\hbar^2/(2mL^2).$$

Prepare $\psi = (\psi_1 + i\psi_2)/\sqrt{2}$. Orthogonality makes its norm one. Energy measurements give E1 or 4E1 with equal probability:

$$\langle E\rangle = \tfrac{1}{2}E_1 + \tfrac{1}{2}(4E_1) = \tfrac{5}{2}E_1.$$

This average is not an allowed single energy result. The relative phase changes in time, so a superposition of different energies is not a stationary state.



2 Practice

2.1 Choose one answer. In an infinite well, E3/E1 is: [1]

- **A** 3
- **B** 6
- **C** 9
- **D** The boundary conditions determine no energies
- **E** Replace $\hbar$ with h without changing the coefficient

2.2 Choose one answer. The probability density is: [1]

- **A** ψ
- **B** $|\psi|^2$
- **C** the energy alone
- **D** The boundary conditions determine no energies
- **E** Replace $\hbar$ with h without changing the coefficient

2.3 If E1=2 eV, E2=_____ eV. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: A normalised quantum state has total probability one. [2]

3 Exam-style questions

3.1 In the same well a normalised state is $(\psi_1 + 2\psi_3)/\sqrt{5}$. Find its possible measured energies and probabilities, mean energy and energy variance. Is the mean an allowed result of one measurement? [5]