

42.1 Simultaneity, length measurement and velocity transformation

Name: _____ Class: _____ Date: _____

Total: 10 marks**KEY WORDS** spacetime interval • proper length

Objective

Transform event separations and identify proper length versus simultaneous endpoint readings; Apply relativistic velocity addition and check a four-vector invariant; Classify the interval sign to order events and rule out causal contact. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Transform separations with $x'=\gamma(x-vt)$, $t'=\gamma(t-vx/c^2)$. Events simultaneous in S with $\Delta x \neq 0$ are not simultaneous in S': $\Delta t' = -\gamma v \Delta x / c^2$.

A frame length uses simultaneous endpoint readings in that frame; then $L=L_0/\gamma$. Different reading conditions answer different questions.

$s^2=c^2\Delta t^2-\Delta x^2$ is invariant: timelike separations have a co-located frame and fixed time order; spacelike separations have a simultaneous frame and can swap order.

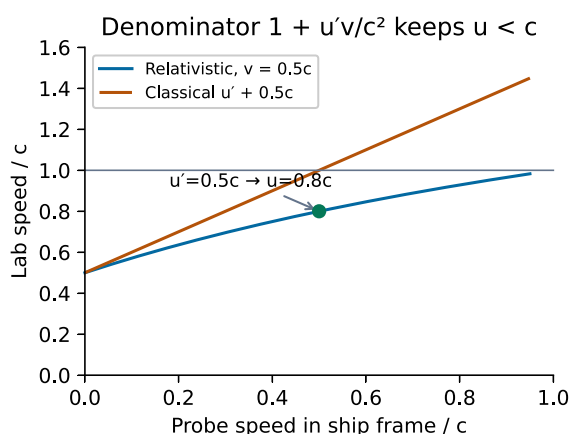
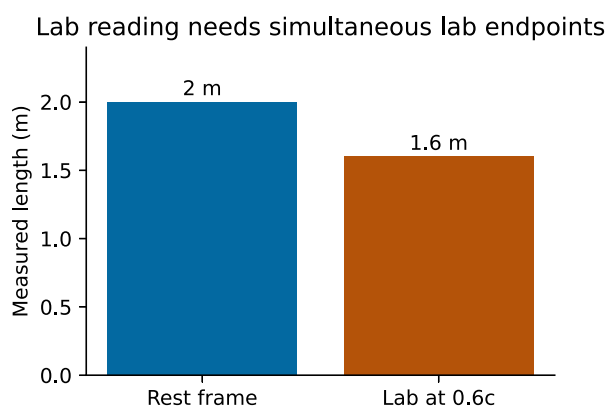
$u=(u'+v)/(1+u'v/c^2)$ keeps speeds below c; $E^2-p^2c^2=m^2c^4$ is frame-independent. Check the nonrelativistic limit.

■ Worked method

Two lab events have $\Delta x = 900$ m and $\Delta t = 2$ microseconds. Their spacetime interval is spacelike because $c \Delta t = 600$ m is less than Δx . A frame making them simultaneous satisfies

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0, \quad v = c^2\Delta t/\Delta x = 2c/3.$$

Their separation in that frame is $\sqrt{\Delta x^2 - c^2\Delta t^2} = 670.8$ m. No frame can put these spacelike events at the same position.



2 Practice

2.1 Choose one answer. To obtain the contracted length of a moving rod, a lab observer must record the two endpoint positions: [1]

- **A** simultaneously in the rod rest frame
- **B** simultaneously in the lab frame
- **C** at the same lab position at different times
- **D** at equal rod-frame times only
- **E** without any simultaneity condition

2.2 Choose one answer. A ship moves at $0.5c$ and launches a probe forward at $0.5c$ relative to the ship. The probe has lab speed: [1]

- **A** $0.75c$
- **B** $0.8c$
- **C** $0.9c$
- **D** $1.0c$
- **E** exactly c

2.3 Two lab events have $\Delta x = 900$ m and $\Delta t = 2 \mu s$. Using $c = 3 \times 10^8$ m/s, the frame in which they are simultaneous moves at _____ c (give the exact fraction). Show your relation and units. [2]

