4.1 Thermodynamics and statistical mechanics

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS entropy • partition function

Objective

Apply the first and second laws with a sign convention; Relate ensembles, Boltzmann factors and entropy ; Analyse ideal gases, heat engines and quantum statistics. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

State the first-law sign convention: ΔU=Q−W when W is work done by the system.

For a reversible engine between reservoirs, efficiency is bounded by 1−Tc/Th using absolute temperatures.

Boltzmann weights are exp(−E/kT). Probabilities require division by the partition function .

Distinguish Maxwell–Boltzmann, Bose–Einstein and Fermi–Dirac assumptions. Fermions cannot share an identical single-particle state.

■ Worked method

Use work W done by the gas as positive. The first law is

$$\Delta U = Q - W.$$

For reversible isothermal expansion of an ideal gas, U depends only on T, so

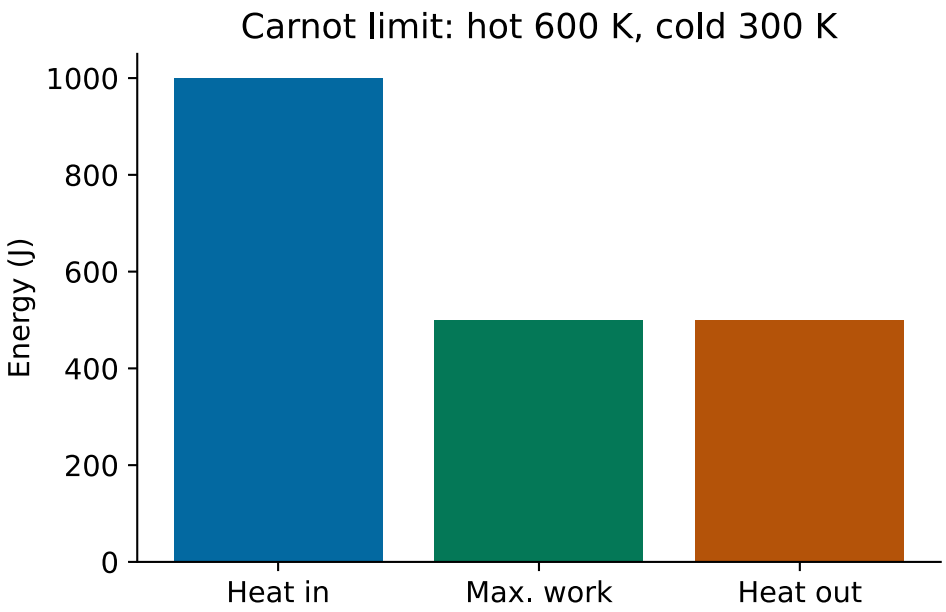
$$W = \int_{V_i}^{V_f} p\,dV = nRT\ln(V_f/V_i), \qquad Q = W.$$

For n = 0.50 mol, T = 300 K and doubled volume,

$$W = nRT\ln 2 = (0.50\,\text{mol})(8.314\,\text{J/(mol K)})(300\,\text{K})\ln 2 = 864\,\text{J}.$$

$$\Delta S = Q_{rev}/T = (864\,\text{J})/(300\,\text{K}) = 2.88\,\text{J/K}.$$

The gas expands and receives heat. The reservoir loses the same entropy in this reversible limit.



2 Practice

2.1 Choose one answer. A Carnot engine between 400 K and 300 K has efficiency: [1]

- **A** 0.25
- **B** 0.75
- **C** 1.33

2.2 Choose one answer. A higher-energy state's Boltzmann weight at fixed T is: [1]

- **A** greater
- **B** smaller
- **C** always identical

2.3 A system receives 100 J heat and does 40 J work. $\Delta U =$_____ J. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: An engine may convert all heat from one reservoir into work in a complete cycle with no other effect. [2]

3 Exam-style questions

3.1 An insulated rigid vessel contains ideal gas confined initially to half its volume. Remove the internal partition into vacuum. Compare Q, W, temperature change and gas entropy change with reversible isothermal expansion to the same final volume. [5]