30.1 RC and RL transients, impedance and resonance

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS time constant • impedance

Objective

Solve RC charging and discharging with stated initial conditions; Solve RL current response and account for stored energy; Use complex impedance , phase and resonance in sinusoidal circuits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For RC, R dq/dt+q/C=V_s and τ=RC. V_C=V_s + (V_C(0)−V_s) exp(−t/τ). An uncharged capacitor reaches 63.2% after τ; discharge leaves 36.8%. Voltage is continuous without impulsive current.

For RL, L dI/dt+RI=V_s and τ=L/R. I = V_s/R + (I(0)−V_s/R) exp(−t/τ). Current is continuous for finite voltage. Stored energy $LI^2/2$ becomes resistor heat during decay.

Using exp(iωt), series RLC has Z=R+iX with X=ωL−1/(ωC). I_rms = V_rms/sqrt(R^2+X^2); current lags for X>0 and leads for X<0. Add complex impedances, not magnitudes.

Series driven resonance has ω0=1/sqrt(LC), X=0 and maximal current. Mean power is I_{rms}^2R. Damped free oscillation instead has frequency $\sqrt{1/(LC) - (R/(2L))^2}$ when underdamped. Distinguish the drive and regime.

■ Worked method

An RC circuit switches to fixed source voltage Vs. Charge conservation and Kirchhoff's law give

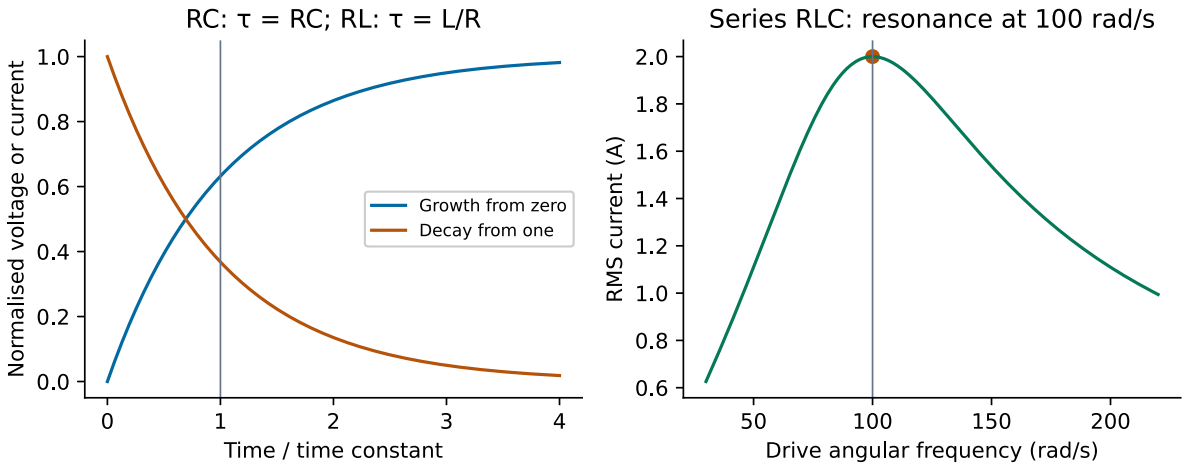
$$RC\dot{V}_C + V_C = V_s, \quad V_C(t) = V_s + [V_C(0) - V_s]e^{-t/(RC)}.$$

For R = 2 kilohms, C = 1 millifarad, Vs = 12 V and initial voltage zero,

$$\tau = RC = (2000\,\Omega)(0.001\,\mathrm{F}) = 2.0\,\mathrm{s}.$$

$$V_C(\tau) = V_s(1 - e^{-1}) = (12\,\mathrm{V})(1 - e^{-1}) = 7.59\,\mathrm{V}.$$

The time constant controls the remaining difference from the final state.



2 Practice

2.1 Choose one answer. After one time constant, an initially uncharged RC capacitor driven by a constant source reaches approximately: [1]

- **A** 36.8% of final voltage
- **B** 50% of final voltage
- **C** 63.2% of final voltage
- **D** 86.5% of final voltage
- **E** 100% of final voltage

2.2 Choose one answer. Using $\exp(i\omega t)$, a series circuit with $X=\omega L-1/(\omega C)>0$ has: [1]

- **A** Current leading voltage
- **B** Current lagging voltage
- **C** Voltage and current necessarily in phase
- **D** Zero resistor dissipation
- **E** Impedance magnitude $R+X$

2.3 An RL circuit has $L=0.6$ H and $R=3\ \Omega$. Its time constant is _____ s. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: An ideal inductor stores energy but dissipates no average real power in sinusoidal steady state. [2]

3 Exam-style questions

3.1 An RC capacitor starts at 8 V and switches to a -4 V ideal source through R. Find its voltage at arbitrary t, the first zero crossing and the initial current defined into its positive-voltage plate. Explain voltage continuity. [5]