

27.1 Accelerating and rotating reference frames

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS Coriolis force • centrifugal force

Objective

Transform acceleration with translating and rotating frame terms; Determine centrifugal, Coriolis and Euler directions from cross products; Separate apparent forces from real interactions and test inertial limits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

A translating origin with acceleration A adds $-mA$ to the frame force equation: $m a' = F_{\text{real}} - mA$. For an upward-accelerating elevator, $N = m(g+a)$. A uniformly translating frame adds no apparent force.

Rotating axes give $a = a_{\text{origin}} + a' + 2\Omega \times v' + \Omega \times (\Omega \times r) + \dot{\Omega} \times r$. Moving the extra terms to the force side gives Coriolis, centrifugal and Euler apparent forces. Use relative v' and one consistent vector basis.

For Ω along $+z$, r and outward v' along $+x$, centrifugal force points $+x$ while Coriolis points $-y$. Reversing v' reverses Coriolis. F_C is perpendicular to v' and does no instantaneous relative work.

A body fixed on a constant-speed platform has $v' = a' = 0$; its real inward force cancels the outward centrifugal term. Constant Ω removes Euler only. The $\Omega \rightarrow 0$ and $A \rightarrow 0$ limits recover ordinary Newtonian motion.

■ Worked method

In a rotating frame, Coriolis force uses the relative velocity.

$$\mathbf{F}_C = -2m\boldsymbol{\Omega} \times \mathbf{v}', \quad \mathbf{F}_{cf} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$$

For $m = 2 \text{ kg}$, $\Omega = 2 \text{ rad/s}$ along z , $r = 1 \text{ m}$ along x and $v' = 3 \text{ m/s}$ along x ,

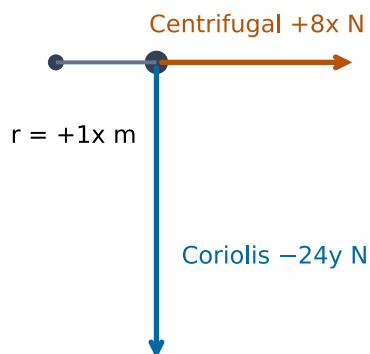
$$F_{cf} = m\Omega^2 r = (2 \text{ kg})(2 \text{ rad/s})^2(1 \text{ m}) = 8 \text{ N} \quad (+x).$$

$$F_C = 2m\Omega v' = 2(2 \text{ kg})(2 \text{ rad/s})(3 \text{ m/s}) = 24 \text{ N} \quad (-y).$$

Euler force is zero only because Ω is constant. These are coordinate terms, not extra physical contacts.

Constant anticlockwise frame; apparent forces

Rotation axis; $\Omega = +2\hat{z}$



$m = 2\text{ kg}$; $v' = +3\hat{x}\text{ m/s}$; arrows show directions

2 Practice

2.1 Choose one answer. Ω points $+\hat{z}$ and v' points $+\hat{x}$. The Coriolis force points: [1]

- **A** $+\hat{x}$
- **B** $-\hat{x}$
- **C** $+\hat{y}$
- **D** $-\hat{y}$
- **E** $+\hat{z}$

2.2 Choose one answer. A mass stays fixed at nonzero radius on a platform with constant rotation. Which statement is correct? [1]

- **A** Its inertial acceleration is zero
- **B** Its Coriolis force is nonzero because the platform turns
- **C** A real inward force balances the rotating-frame centrifugal term
- **D** Centrifugal force is an additional real interaction
- **E** Its real force must be outward

2.3 A 3 kg passenger accelerates downward at 2 m/s^2 with $g=10$. The scale force is _____ N. Show your relation and units. [2]

