

## 20.1 Speed distributions and statistical ensembles

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 10 marks

KEY WORDS probability density • partition function

### Objective

Normalise continuous speed densities and interpret their units; Distinguish vector means, speed means and most probable speeds; Use Boltzmann weights with degeneracy and a partition function . Original public practice; not an ETS paper.

### 1 Key knowledge and worked examples

A speed probability density  $f$  has  $\int f(v)dv=1$  and inverse-speed units. A count density  $P$  has area  $N$  and  $f=P/N$ . A trapezoid rising over  $v_0$ , flat over  $2v_0$  and falling over  $v_0$  has area  $3av_0$  for height  $a$ . Normalise by area, not height.

Without bulk flow an isotropic gas has zero mean velocity but positive mean speed. Maxwell speed density is proportional to  $v^2 \exp[-mv^2/(2k_B T)]$ . Mode, mean and rms speeds are  $\sqrt{2k_B T/m}$ ,  $\sqrt{8k_B T/(\pi m)}$  and  $\sqrt{3k_B T/m}$ . The  $v^2$  factor makes speed density zero at  $v=0$ .

For a continuous distribution, an exact single speed has probability zero; intervals have nonzero probabilities. This does not exclude arbitrarily slow particles or a detector's finite-width zero bin. Characteristic speeds scale as  $\sqrt{T/m}$ , not  $T$ .

Canonical state weights are  $\exp(-E/(k_B T))$ ; divide by their sum  $Z$ . Level probability includes degeneracy  $g$ :  $g \exp(-E/(k_B T))/Z$ . Ground degeneracy 1 and excited degeneracy 2 at gap  $k_B T \ln 2$  give equal level weights. Classical Maxwell–Boltzmann differs from quantum Bose/Fermi occupations.

#### ■ Worked method

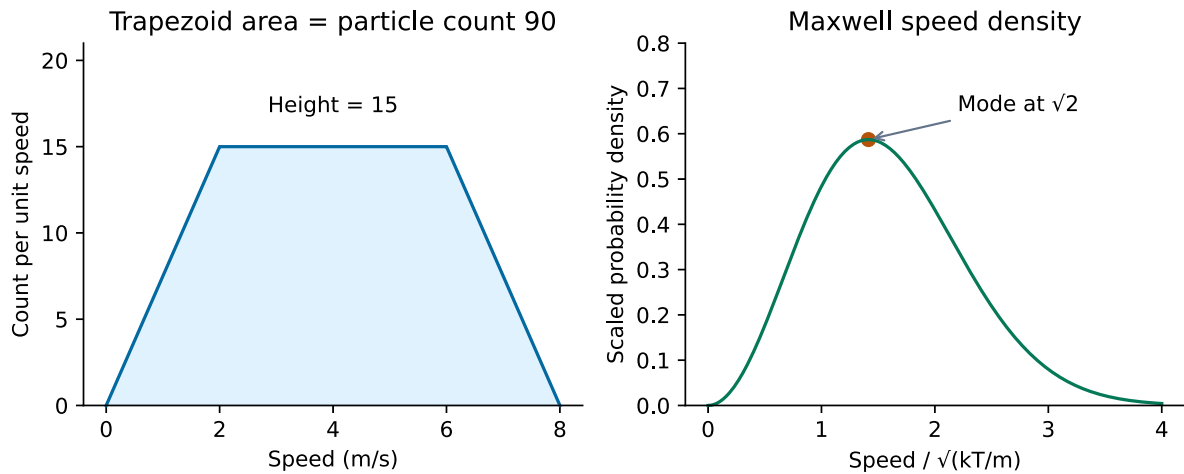
A canonical ensemble has ground degeneracy 1 and excited degeneracy  $g$  at gap  $\Delta$ .

$$Z = 1 + g e^{-\Delta/(k_B T)}, \quad P_e = \frac{g e^{-\Delta/(k_B T)}}{Z}.$$

With  $g = 4$  and  $\Delta = k_B T \ln 2$ ,

$$P_e = \frac{4e^{-\ln 2}}{1 + 4e^{-\ln 2}} = \frac{2}{3}.$$

This is probability for the entire excited level. Each of its four states has probability  $1/6$ .



## 2 Practice

**2.1** Choose one answer. For an isotropic equilibrium ideal gas without bulk flow, which is correct? [1]

- **A** Mean velocity and mean speed are both zero
- **B** Mean velocity is zero but mean speed is positive
- **C** The most probable speed is zero
- **D** All particles have the rms speed
- **E** Speed density has no units

**2.2** Choose one answer. Ground degeneracy 1, excited degeneracy 3, gap  $kBT \ln 3$ . The excited-level probability is: [1]

- **A**  $1/4$
- **B**  $1/3$
- **C**  $1/2$
- **D**  $3/4$
- **E** 1

**2.3** A count-density trapezoid has area  $3av_0$ ,  $N=60$  and  $v_0=4$  m/s. Its height  $a$  is \_\_\_\_\_. Show your relation and units. [2]

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