16.1 Acoustic Doppler echoes and standing-wave boundaries

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS displacement antinode • Doppler factor

Objective

Apply source and observer Doppler factors in the medium frame; Derive allowed pipe frequencies from displacement boundary conditions; Distinguish frequency, wavelength and boundary changes. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For sound in a stationary medium, approaching source speed u_s gives f_received=f c/(c−u_s). Approaching observer speed u_o gives f_received=f(c+u_o)/c. Source motion changes wavefront spacing; observer motion changes encounter rate. These roles are not interchangeable; use medium-frame velocities and reverse signs for recession.

A stationary wall receives f c/(c−u), then reflects that frequency unchanged in the medium frame. The approaching driver receives another factor (c+u)/c, giving f_echo=f(c+u)/(c−u). At small u/c the fractional shift is about 2u/c; a moving reflector needs its own analysis.

Ideal open ends have displacement antinodes/pressure nodes; closed ends have displacement nodes/pressure antinodes. Both-open modes have f_n=nc/(2L). One-closed modes have f_n=(2n−1)c/(4L). These formulas assume ideal boundaries; real end corrections require information not supplied by the simple model.

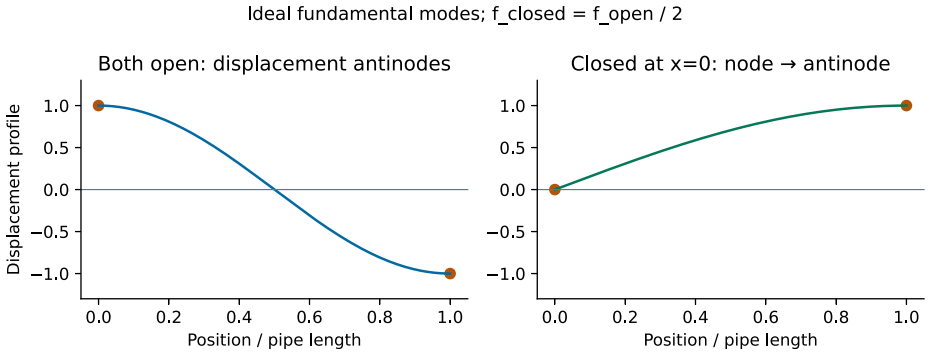
Closing one end halves the fundamental and permits only odd multiples of that new value. Old both-open lines are even multiples of the new fundamental, so none remains in the ideal closed spectrum. Medium wave speed stays unchanged while frequency and wavelength adapt to the new boundaries.

■ Worked method

A siren approaches a stationary wall in still air. Treat outward and return journeys separately. The wall first receives $f_w = fc/(c-u)$; reflection keeps this frequency in the medium frame. The moving listener then receives $f_e = f_w(c+u)/c$.

$$f_e = f\frac{c+u}{c-u} = (600\,\text{Hz})\frac{340\,\text{m/s}+20\,\text{m/s}}{340\,\text{m/s}-20\,\text{m/s}} = 675\,\text{Hz}.$$

This Doppler effect uses velocities relative to the medium. It is not the light-wave formula.



2 Practice

2.1 Choose one answer. A moving siren approaches a stationary wall at speed u in still air with speed c. The driver's echo frequency divided by emitted frequency is: [1]

- **A** c/(c−u)
- **B** (c+u)/c
- **C** (c+u)/(c−u)
- **D** (c−u)/(c+u)
- **E** 1

2.2 Choose one answer. An ideal both-open pipe has fundamental 120 Hz. After closing one end, its first three allowed frequencies are: [1]

- **A** 120,240,360 Hz
- **B** 120,360,600 Hz
- **C** 60,120,180 Hz
- **D** 60,180,300 Hz
- **E** 240,480,720 Hz

2.3 For c=330 m/s and u=30 m/s toward a stationary wall, f_echo/f=_____ (decimal). Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Closing one end of an ideal both-open pipe retains its original fundamental as the new second harmonic. [2]

3 Exam-style questions

3.1 A stationary source and receiver face a reflector approaching at speed u in still air. Derive the reflected frequency and invert the relation to find u from measured ratio r. Check the low-speed limit. [4]