15.1 Maxwell waves, energy flow and boundary conditions

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS displacement current • Poynting vector

Objective

Use displacement current and material wave-speed relations; Determine wave-field directions and radiated energy flow; Apply Maxwell boundary conditions without confusing field components. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Vacuum Ampere–Maxwell law is $\oint$B · dl=μ0(I+ε0dΦE/dt). Displacement current depends on changing electric flux, not magnetic flux. It makes capacitor-loop results consistent for surfaces cutting the wire or crossing the gap. For a uniform gap field its value is ε0A dE/dt.

In a homogeneous linear lossless medium, v=1/sqrt($\mu\varepsilon$), n=sqrt(μrεr). For nonmagnetic material, v=c/sqrt(εr). Use constants appropriate to the frequency if dispersion matters. E/B=v in this simple medium, and c in vacuum.

For propagation unit vector n, B=(n×E)/v. Phase kz−ωt travels +z; kz+ωt travels −z. A +z wave with E along x+y has B along −x+y. Poynting vector E×B/μ gives energy-flow direction; far-zone radiation from an accelerating charge carries energy outward.

Because ∇ · B=0, normal B is continuous across a boundary. Zero interior B in an ideal Meissner model therefore gives zero exterior normal B; a tangential field can remain with a surface current. This conclusion uses the specified model, not an assumption of zero field everywhere. Normal D can instead jump by free surface charge.

■ Worked method

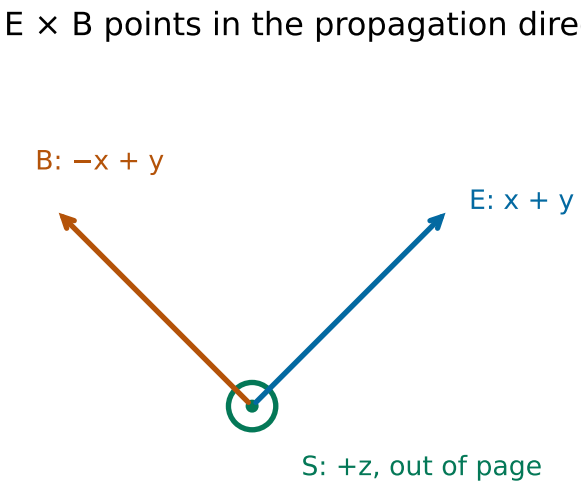
A plane wave travels along positive z in a linear lossless nonmagnetic dielectric . With relative permittivity 9,

$$v = c/\sqrt{\epsilon_r} = (3.00 \times 10^8\,\mathrm{m/s})/3 = 1.00 \times 10^8\,\mathrm{m/s}.$$

If E points along positive x, $\mathbf{B} = \hat{\mathbf{z}} \times \mathbf{E}/v$ points along positive y. For E0 = 200 V/m,

$$B_0 = E_0/v = (200\,\mathrm{V/m})/(1.00 \times 10^8\,\mathrm{m/s}) = 2.00\,\mu\mathrm{T}.$$

$\mathbf{E} \times \mathbf{H}$ points along propagation. Check that material constants apply at the wave frequency.



2 Practice

2.1 Choose one answer. For a +z plane wave whose electric field is along +x, magnetic field points: [1]

- **A** +x
- **B** −x
- **C** +y
- **D** −y
- **E** +z

2.2 Choose one answer. If interior B is zero at an ideal Meissner boundary, a nonzero outside B must be: [1]

- **A** Normal to the surface
- **B** Tangent to the surface
- **C** Zero at all outside points
- **D** Time dependent
- **E** Parallel to E in every case

2.3 In a nonmagnetic lossless dielectric with εr=16, wave speed is _____ times c (decimal). Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: For capacitor charging, displacement current depends on the rate of change of electric flux rather than magnetic flux. [2]

3 Exam-style questions

3.1 A vacuum plane wave travels along negative x, with electric field along positive z. Determine magnetic-field direction and average energy flux for peak electric amplitude E0. A perfect absorber of area A is normal to the beam. Find its mean force. [4]