13.1 Electrostatic superposition, flux and conductors

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS solid angle • induced surface charge

Objective

Add Coulomb forces as vectors using the actual geometry; Compute flux from symmetry or solid angle with a stated orientation; Find conductor fields and potentials using induced surface charges. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Point-charge fields and Coulomb forces add as vectors. Same-direction components add, opposite components subtract, and perpendicular components combine by Pythagoras. Potential instead adds as a scalar, V=ΣQ/(4πε0r), with a stated reference. Zero potential does not generally imply zero field: E=−∇V.

Gauss's law ∮E · dA=Q_enclosed/ε0 concerns a closed surface. Symmetry is needed to infer a simple local field. An infinite open plane subtends solid angle 2π, giving half a point charge's total flux in magnitude. Its sign depends on the normal; the plane does not enclose the charge.

For centred charge Q inside a conducting spherical shell of net charge q, equilibrium zero field in the material induces inner charge −Q and outer charge Q+q. With outer radius b and zero potential at infinity, the material potential is (Q+q)/(4πε0b), independent of observation radius.

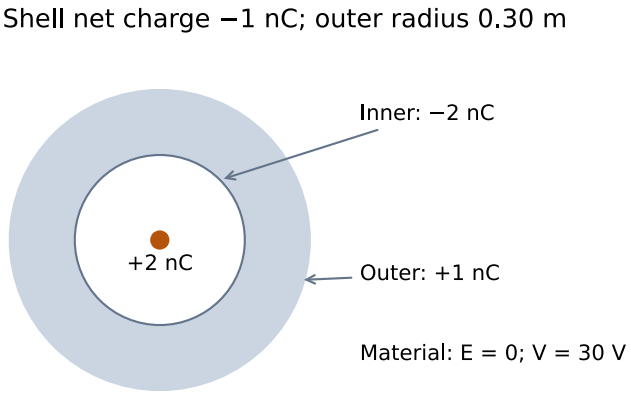
In the cavity, a centred charge gives radial field Q/(4πε0r²). Potential includes constant shell contributions and is continuous across each surface; the normal field can jump. Off-centre Q still induces total inner charge −Q, but the cavity field is no longer the centred radial expression.

■ Worked method

A centred point charge Q lies in a conducting spherical shell of net charge q. Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{inner} = -Q$. Charge conservation gives $Q_{outer} = q+Q$. With outer radius b and zero potential at infinity,

$$V_{metal} = \frac{q+Q}{4\pi\epsilon_0 b} = \frac{(9.0\times10^{9}\,\mathrm{N\,m^2/C^2})(1.0\times10^{-9}\,\mathrm{C})}{0.30\,\mathrm{m}} = 30\,\mathrm{V}.$$

Here Q = +2 nC and q = -1 nC. The cavity is not conducting material; its field need not vanish.



2 Practice

2.1 Choose one answer. A centred charge +3Q lies inside a shell whose net charge is −Q. Its outer-surface charge is: [1]

- **A** −3Q
- **B** −Q
- **C** 0
- **D** +2Q
- **E** +3Q

2.2 Choose one answer. Two forces of magnitudes 6 N and 8 N are perpendicular. Their resultant magnitude is: [1]

- **A** 2 N
- **B** 7 N
- **C** 10 N
- **D** 14 N
- **E** 48 N

2.3 The magnitude of flux from a positive point charge through an infinite plane equals _____ times $Q/\varepsilon 0$ (decimal). Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Inside equilibrium conducting material, nonzero constant potential is compatible with zero electric field. [2]

3 Exam-style questions

3.1 Move Q away from the centre while keeping it inside the closed cavity. The shell's exterior remains a sphere, with no external charges. Which of inner charge total, inner charge distribution, metal potential and exterior field retain their centred values? Explain using Gauss's law and electrostatic uniqueness. [5]