10.1 Collisions, work and oscillator energy

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS external impulse • turning point

Objective

Compute vector momentum and energy loss in sticking collisions; Compare fixed-force springs and use work–energy conditions; Derive small-oscillation periods and oscillator energies. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

With negligible external impulse , conserve momentum in each direction. For sticking masses, v=(m1v1+m2v2)/(m1+m2). Add vectors before taking magnitudes. Compare Σ½m|v|^2 initially with ½(m1+m2)|v|^2 finally; the loss becomes internal energy/deformation. Sticking does not mean final rest.

Work–energy gives ΔK=∫F · dr, or Fs for constant parallel force. Mechanical-energy conservation requires no unaccounted nonconservative work. Spring energy is ½kx^2. At fixed force, x=F/k and U=F^2/(2k); at fixed extension U increases with k. These comparisons have opposite answers.

Near stable equilibrium, U≈U0+½k_eff(x−x0)^2 with k_eff=U″(x0)>0. Then ω=sqrt(k_eff/m), T=2π/ω. A small-angle pendulum uses sinθ≈θ in θddot+(g/L)sinθ=0, giving T=2πsqrt(L/g). Large amplitudes need a correction; ideal pendulum mass cancels.

Undamped oscillator energy is ½mv^2+½kx^2=½kA^2. At equilibrium it is all kinetic; at turning points it is all potential. Use ω=2πf and A=vmax/ω. The measurement position determines which energy expression is simplest; convert joules to millijoules only after calculation.

■ Worked method

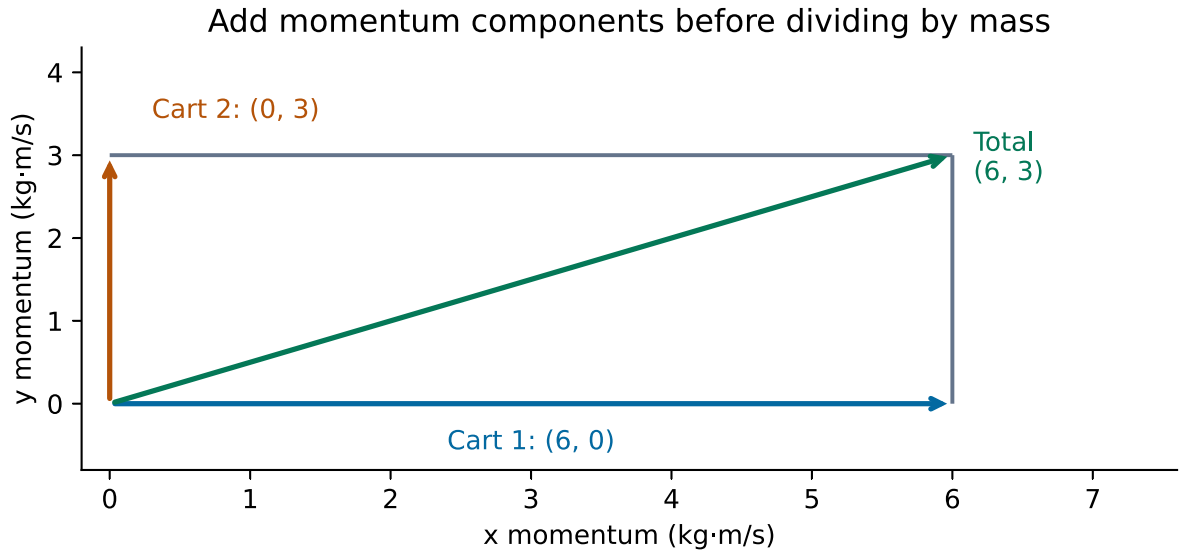
Two carts stick with negligible external impulse. Add momentum vectors , then divide by total mass. Known: m1 = 2 kg, v1 = (3,0) m/s; m2 = 1 kg, v2 = (0,3) m/s.

$$\mathbf{v}_f = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{(2\,\text{kg})(3,0)\,\text{m/s} + (1\,\text{kg})(0,3)\,\text{m/s}}{3\,\text{kg}} = (2,1)\,\text{m/s}.$$

$$K_i = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(2\,\text{kg})(3\,\text{m/s})^2 + \tfrac{1}{2}(1\,\text{kg})(3\,\text{m/s})^2 = 13.5\,\text{J}.$$

$$K_f = \tfrac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 = \tfrac{1}{2}(3\,\text{kg})(5\,\text{m}^2/\text{s}^2) = 7.5\,\text{J}.$$

The 6 J difference becomes internal energy. Kinetic energy is not conserved in sticking.



2 Practice

2.1 Choose one answer. For the two carts in the worked example, the common velocity is: [1]

- **A** (3,3) m/s
- **B** (2,1) m/s
- **C** (1,2) m/s
- **D** (2,2) m/s
- **E** (0,0) m/s

2.2 Choose one answer. Two springs with k1=3k2 hold the same force. The ratio U1/U2 is: [1]

- **A** 3
- **B** 9
- **C** 1
- **D** 1/3
- **E** 1/9

2.3 A small-angle pendulum length is multiplied by 9 at fixed g. Its period is multiplied by _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: At equilibrium an undamped harmonic oscillator can have nonzero kinetic energy and zero spring potential energy. [2]

3 Exam-style questions

3.1 Two equal masses approach with speed u along directions separated by angle theta and stick. Derive final speed and the fraction of initial kinetic energy lost. Check parallel, perpendicular and opposite incoming directions. [5]