

42.1 Simultaneity, length measurement and velocity transformation

Name: _____ Class: _____ Date: _____

Total: 10 marks**KEY WORDS** spacetime interval • proper length

Objective

Transform event separations and identify proper length versus simultaneous endpoint readings; Apply relativistic velocity addition and check a four-vector invariant; Classify the interval sign to order events and rule out causal contact. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Transform separations with $x' = \gamma(x - vt)$, $t' = \gamma(t - vx/c^2)$. Events simultaneous in S with $\Delta x \neq 0$ are not simultaneous in S': $\Delta t' = -\gamma v \Delta x / c^2$.

A frame length uses simultaneous endpoint readings in that frame; then $L = L_0 / \gamma$. Different reading conditions answer different questions.

$s^2 = c^2 \Delta t^2 - \Delta x^2$ is invariant: timelike separations have a co-located frame and fixed time order; spacelike separations have a simultaneous frame and can swap order.

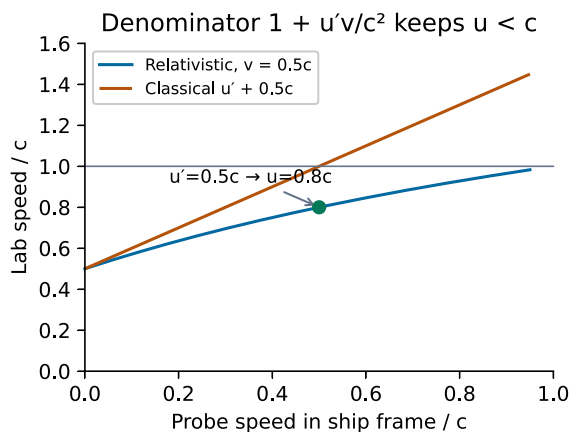
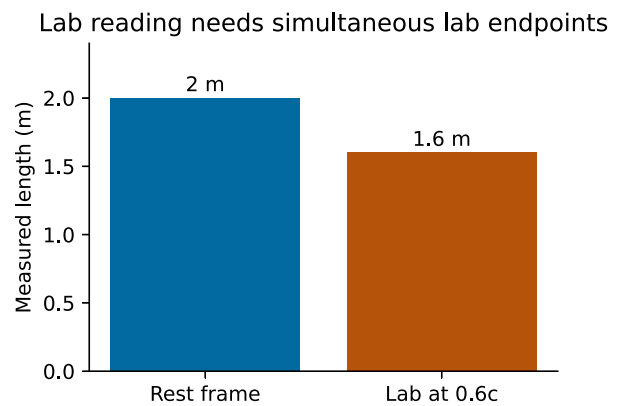
$u = (u' + v) / (1 + u'v/c^2)$ keeps speeds below c; $E^2 - p^2 c^2 = m^2 c^4$ is frame-independent. Check the nonrelativistic limit.

■ Worked method

Two lab events have $\Delta x = 900$ m and $\Delta t = 2$ microseconds. Their spacetime interval is spacelike because $c \Delta t = 600$ m is less than Δx . A frame making them simultaneous satisfies

$$\Delta t' = \gamma(\Delta t - v \Delta x / c^2) = 0, \quad v = c^2 \Delta t / \Delta x = 2c/3.$$

Their separation in that frame is $\sqrt{\Delta x^2 - c^2 \Delta t^2} = 670.8$ m. No frame can put these spacelike events at the same position.



2 Practice

2.1 Choose one answer. To obtain the contracted length of a moving rod, a lab observer must record the two endpoint positions: [1]

- **A** simultaneously in the rod rest frame
- **B** simultaneously in the lab frame
- **C** at the same lab position at different times
- **D** at equal rod-frame times only
- **E** without any simultaneity condition

2.2 Choose one answer. A ship moves at $0.5c$ and launches a probe forward at $0.5c$ relative to the ship. The probe has lab speed: [1]

- **A** $0.75c$
- **B** $0.8c$
- **C** $0.9c$
- **D** $1.0c$
- **E** exactly c

2.3 Two lab events have $\Delta x = 900$ m and $\Delta t = 2 \mu s$. Using $c = 3 \times 10^8$ m/s, the frame in which they are simultaneous moves at _____ c (give the exact fraction). Show your relation and units. [2]

Solutions

2.1 B Length in a frame is the endpoint separation at equal time in that frame; $\Delta t=0$ in the lab gives $L=L_0/\gamma$.

Compare the rejected options: simultaneously in the rod rest frame; at the same lab position at different times; at equal rod-frame times only; without any simultaneity condition. Use the stated model and conditions.

2.2 B $(0.5+0.5)/(1+0.25)=0.8$; the denominator keeps the result below c .

Compare the rejected options: $0.75c$; $0.9c$; $1.0c$; exactly c . Use the stated model and conditions.

2.3 $2/3$. Set $\Delta t'=0$ in $t'=\gamma(t-vx/c^2)$: $v=c^2\Delta t/\Delta x=(600/900)c=2c/3$.

2.4 False. Spacelike order can swap between frames; only timelike or lightlike order is invariant.

3.1 Set $\Delta x' = \gamma(\Delta x - v\Delta t) = 0$, so $v = \Delta x/\Delta t = 0.5c$. The proper time is $\sqrt{\Delta t^2 - (\Delta x/c)^2} = \sqrt{3}$ microseconds. They are timelike separated, and every time-oriented inertial frame retains their order. Award velocity condition, speed, proper time and causal classification/order.