34.1 Thermal transport, calorimetry and expansion

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS thermal resistance • latent heat

Objective

Combine conductive thermal resistances and distinguish heat rate from heat flux; Balance sensible and latent heat with stated isolation and heat-capacity conditions; Calculate free thermal expansion and constrained thermal stress with their limits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Fourier flux is $-\kappa$ dT/dx; steady slab rate κAΔT/L. R_th=L/(κA) in K/W; add series R or parallel 1/R. Rate is W; flux is W/m^2.

With constant specific heat, Q=mcΔT. Sum all energy changes for isolated calorimetry. A phase change needs Q=mℓ at the transition temperature; sensible and latent energy are distinct.

Free isotropic small expansion: ΔL/L≈αΔT, ΔA/A≈2αΔT, ΔV/V≈3αΔT. Prevented elastic expansion gives tensile-signed $\sigma\approx-$YαΔT on heating. State constraints and elastic limits.

Convection moves matter; net radiation is $\varepsilon\sigma$_SB A(T$^4-$T_env^4) using kelvin. Lumped Newton cooling gives excess temperature decay exp($-$t/τ), τ=C/(hA), only when internal gradients are negligible.

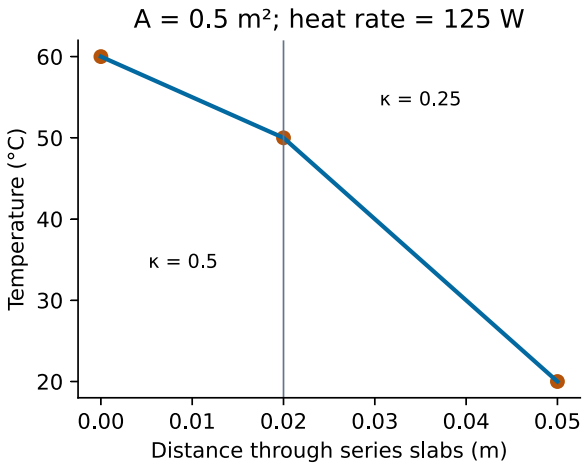
■ Worked method

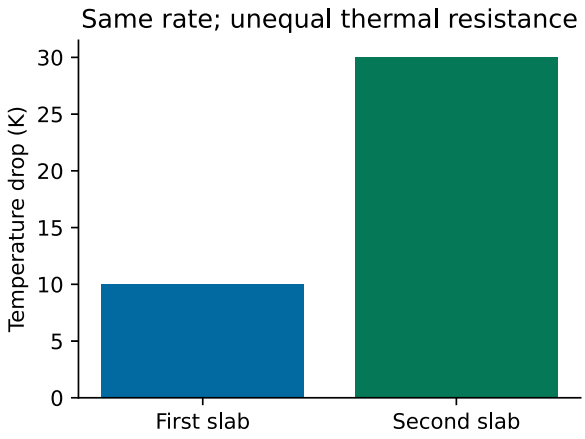
Two slabs in series carry the same steady heat rate. Let resistances be 0.08 and 0.24 K/W. Boundary temperatures are 60 and 20 degrees Celsius.

$$\dot{Q} = \Delta T/(R_1 + R_2) = (40\,\mathrm{K})/(0.32\,\mathrm{K/W}) = 125\,\mathrm{W}.$$

$$\Delta T_1 = \dot{Q}R_1 = (125\,\mathrm{W})(0.08\,\mathrm{K/W}) = 10\,\mathrm{K}.$$

The interface is 50 degrees Celsius when slab 1 touches the hot side. Heat rate has watt units; flux divides it by area.





2 Practice

2.1 Choose one answer. Two slabs in series without sources have thermal resistances R and 3R. The steady temperature-drop ratio is: [1]

- **A** 1:1
- **B** 3:1
- **C** 1:3
- **D** 1:9
- **E** Zero in both

2.2 Choose one answer. An isotropic unconstrained plate with a circular hole is warmed under small uniform expansion. Its hole radius: [1]

- **A** Shrinks
- **B** Stays fixed
- **C** Increases by fractional $\alpha\Delta T$
- **D** Increases by fractional $3\alpha\Delta T$
- **E** Must close

2.3 A body has C=200 J/K and hA=10 W/K under lumped Newton cooling. Its time constant is _____ s. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: A fully constrained elastic rod heated with positive α develops compressive stress if buckling and yield are excluded. [2]

3 Exam-style questions

3.1 Mix 0.20 kg of water at 40 degrees Celsius with 0.050 kg ice at zero degrees Celsius in an insulated container of negligible heat capacity. Use c = 4200 J/(kg K) and latent heat 334000 J/kg. Determine final temperature and whether ice remains. [5]

Solutions

2.1 C Their heat rate is the same; ΔT_i=Qdot R_i gives 1:3.

Compare the rejected options: 1:1; 3:1; 1:9; Zero in both. Use the stated model and conditions.

2.2 C All in-plane lengths, including the hole boundary, scale by $1+\alpha\Delta$T; volume scaling is a different quantity.

Compare the rejected options: Shrinks; Stays fixed; Increases by fractional $3\alpha\Delta$T; Must close. Use the stated model and conditions.

2.3 20. τ=C/(hA)=200/10=20 s.

2.4 True. Mechanical strain must cancel positive thermal strain; tensile-signed stress is $-$Y$\alpha\Delta$T.

3.1 Cooling water to zero releases $mc\Delta T = (0.20)(4200)(40) = 33600$ J. Melting all ice needs $m\ell = (0.050)(334000) = 16700$ J, so all ice melts. Remaining energy heats 0.25 kg water: $T_f = (33600 - 16700)/(0.25 \times 4200) = 16.10$ degrees Celsius. Award cooling budget, melt budget, phase decision, final heat capacity and temperature. If cooling energy were insufficient, final temperature would be zero with some ice left.