32.1 Diffraction envelopes and missing interference orders

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS diffraction envelope • missing order

Objective

Derive and evaluate the far-field single-slit intensity using amplitude superposition; Separate double-slit interference spacing from finite-aperture envelope width; Identify missing orders and state the limits of far-field and small-angle formulas. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Uniform far-field slit amplitude is $\sin\beta/\beta$, $\beta=\pi a \sin\theta/\lambda$. Square it for intensity and take its centre limit as one. Zeros have a $\sin\theta=m\lambda$, $m\neq0$.

For small angles, first-zero displacement is $L\lambda/a$ and central width $2L\lambda/a$. Use $\theta=\text{asin}(m\lambda/a)$, $y=L\tan\theta$ for larger angles; the far-field assumption remains necessary.

Two coherent equal finite slits give normalised $I=(\sin\beta/\beta)^2\cos^2\alpha$, $\alpha=\pi d \sin\theta/\lambda$. Interference spacing $\approx\lambda L/d$; envelope width $\approx 2\lambda L/a$. Width and separation have different roles.

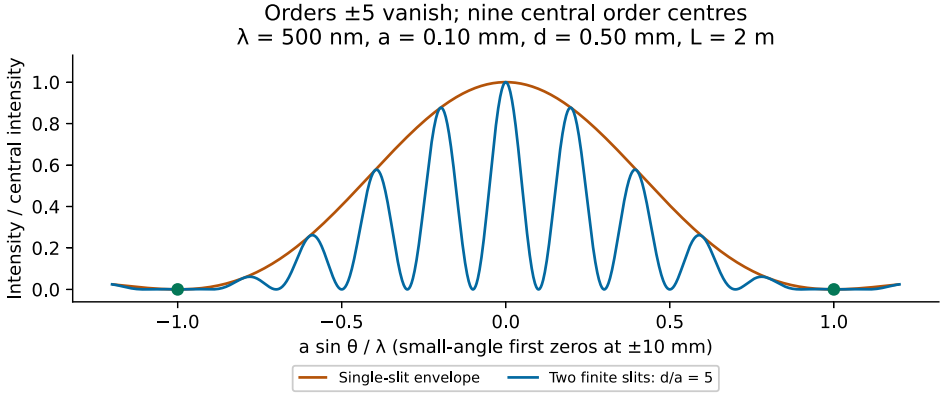
Missing orders satisfy $n=m\ d/a$. Integer $d/a=r$ cancels $\pm r$, $\pm 2r$, etc.; central-envelope nominal order centres satisfy $|n|<r$, giving $2r-1$. Exact product maxima can be slightly shifted.

■ Worked method

For two finite slits, separate interference spacing from the single-slit envelope.

$$I/I_{centre} = (\sin\beta/\beta)^2\cos^2\alpha, \quad \beta = \pi a \sin\theta/\lambda, \quad \alpha = \pi d \sin\theta/\lambda.$$

If $d = 5a$, the first envelope zero $a\sin\theta = \lambda$ coincides with order $n = 5$. Nominal order centres in the central envelope are $n = -4$ through $+4$: nine in total. These are interference centres; the envelope can shift the exact product maxima slightly.



2 Practice

2.1 Choose one answer. Two equal finite slits have d=4a. Which nominal interference orders are cancelled by the first single-slit minima? [1]

- **A** ±1
- **B** ±2
- **C** ±3
- **D** ±4
- **E** Only 0

2.2 Choose one answer. Double slit width a is doubled while d, λ and L stay fixed in the small-angle far field. What changes? [1]

- **A** Spacing and envelope both double
- **B** Fringe spacing halves; envelope fixed
- **C** Fringe spacing fixed; envelope width halves
- **D** Envelope doubles; fringe spacing fixed
- **E** All intensity zeros disappear

2.3 With λ=600 nm, L=1 m and a=0.20 mm, small-angle central single-slit width is _____ mm. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: The single-slit centre is dark because $\sin\beta/\beta$ becomes $0/0$ there. [2]

3 Exam-style questions

3.1 A double slit has d = 3a/2. Which positive interference orders are missing? How many nominal interference centres lie strictly between the first envelope zeros? Explain why rounding d/a to an integer is invalid. [4]

Solutions

2.1 D d $\sin\theta$=nλ and a $\sin\theta$=$\pm\lambda$ coincide when n=$\pm$d/a=$\pm$4.

Compare the rejected options: $\pm$1; $\pm$2; $\pm$3; Only 0. Use the stated model and conditions.

2.2 C Spacing λL/d is unchanged; central envelope width 2λL/a halves.

Compare the rejected options: Spacing and envelope both double; Fringe spacing halves; envelope fixed; Envelope doubles; fringe spacing fixed; All intensity zeros disappear. Use the stated model and conditions.

2.3 6. 2Lλ/a=0.006 m=6 mm.

2.4 False. The limit is one; the apparent 0/0 is removable, and the centre is the intensity maximum.

3.1 Coincidence requires $n = m(d/a) = 3m/2$ with both m,n nonzero integers. Thus m must be even and missing orders are n = 3,6,9,... . Central centres obey $|n| < 3/2$, so n = -1,0,1: three. Fractional d/a describes actual geometry; rounding changes which zeros coincide. Award integer condition, missing sequence, count and explanation.