

32.1 Diffraction envelopes and missing interference orders

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS diffraction envelope • missing order

Objective

Derive and evaluate the far-field single-slit intensity using amplitude superposition; Separate double-slit interference spacing from finite-aperture envelope width; Identify missing orders and state the limits of far-field and small-angle formulas. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Uniform far-field slit amplitude is $\sin\beta/\beta$, $\beta = \pi a \sin\theta/\lambda$. Square it for intensity and take its centre limit as one. Zeros have a $\sin\theta = m\lambda$, $m \neq 0$.

For small angles, first-zero displacement is $L\lambda/a$ and central width $2L\lambda/a$. Use $\theta = a \sin(m\lambda/a)$, $y = L \tan\theta$ for larger angles; the far-field assumption remains necessary.

Two coherent equal finite slits give normalised $I = (\sin\beta/\beta)^2 \cos^2\alpha$, $\alpha = \pi d \sin\theta/\lambda$. Interference spacing $\approx \lambda L/d$; envelope width $\approx 2\lambda L/a$. Width and separation have different roles.

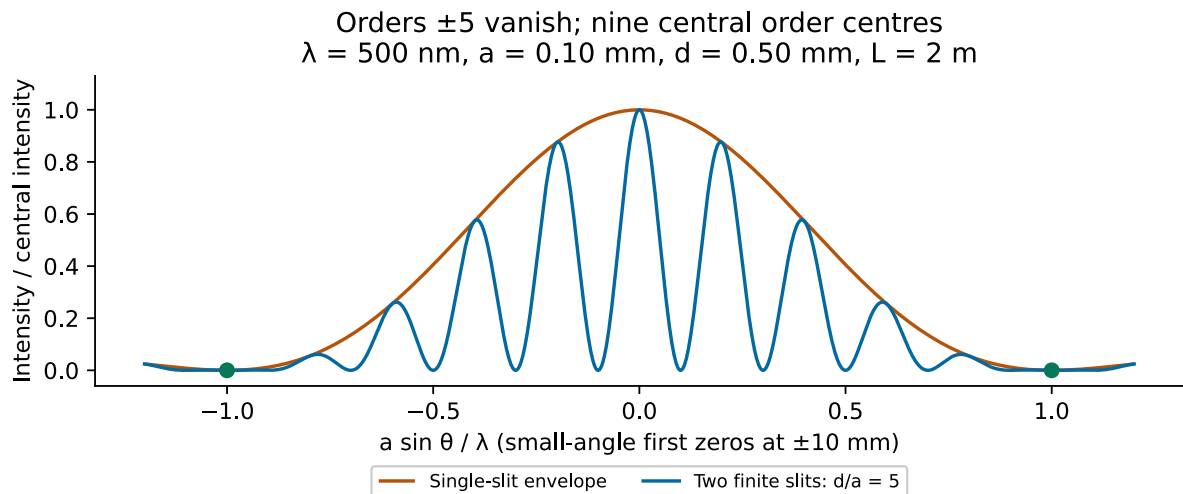
Missing orders satisfy $n = m d/a$. Integer $d/a = r$ cancels $\pm r$, $\pm 2r$, etc.; central-envelope nominal order centres satisfy $|n| < r$, giving $2r - 1$. Exact product maxima can be slightly shifted.

■ Worked method

For two finite slits, separate interference spacing from the single-slit envelope.

$$I/I_{\text{centre}} = (\sin\beta/\beta)^2 \cos^2\alpha, \quad \beta = \pi a \sin\theta/\lambda, \quad \alpha = \pi d \sin\theta/\lambda.$$

If $d = 5a$, the first envelope zero $a \sin\theta = \lambda$ coincides with order $n = 5$. Nominal order centres in the central envelope are $n = -4$ through $+4$: nine in total. These are interference centres; the envelope can shift the exact product maxima slightly.



2 Practice

2.1 Choose one answer. Two equal finite slits have $d=4a$. Which nominal interference orders are cancelled by the first single-slit minima? [1]

- A ± 1
- B ± 2
- C ± 3
- D ± 4
- E Only 0

2.2 Choose one answer. Double slit width a is doubled while d , λ and L stay fixed in the small-angle far field. What changes? [1]

- A Spacing and envelope both double
- B Fringe spacing halves; envelope fixed
- C Fringe spacing fixed; envelope width halves
- D Envelope doubles; fringe spacing fixed
- E All intensity zeros disappear

2.3 With $\lambda=600 \text{ nm}$, $L=1 \text{ m}$ and $a=0.20 \text{ mm}$, small-angle central single-slit width is _____ mm. Show your relation and units. [2]

Solutions

2.1 D $d \sin\theta = n\lambda$ and $a \sin\theta = \pm\lambda$ coincide when $n = \pm d/a = \pm 4$.

Compare the rejected options: ± 1 ; ± 2 ; ± 3 ; Only 0. Use the stated model and conditions.

2.2 C Spacing $\lambda L/d$ is unchanged; central envelope width $2\lambda L/a$ halves.

Compare the rejected options: Spacing and envelope both double; Fringe spacing halves; envelope fixed; Envelope doubles; fringe spacing fixed; All intensity zeros disappear. Use the stated model and conditions.

2.3 6. $2L\lambda/a = 0.006 \text{ m} = 6 \text{ mm}$.

2.4 False. The limit is one; the apparent $0/0$ is removable, and the centre is the intensity maximum.

3.1 Coincidence requires $n = m(d/a) = 3m/2$ with both m, n nonzero integers. Thus m must be even and missing orders are $n = 3, 6, 9, \dots$. Central centres obey $|n| < 3/2$, so $n = -1, 0, 1$: three. Fractional d/a describes actual geometry; rounding changes which zeros coincide. Award integer condition, missing sequence, count and explanation.