31.1 Polarisation, analyser chains and phase

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS Malus's law • Brewster angle

Objective

Calculate successive ideal analyser transmissions from the immediate input state; Distinguish linear, circular and elliptical field motion using relative phase; Use Brewster incidence with refractive-index and plane-of-incidence conventions. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

An ideal analyser projects E onto its axis: amplitude E $\cos\theta$, intensity I $\cos^2\theta$. An ideal first filter passes $I_0/2$ for unpolarised input and makes the output linear.

Update polarisation after every filter. Unpolarised input through axes 0°, 45°, 90° gives $I_0/2$, $I_0/4$, $I_0/8$. A middle filter can restore transmission through an otherwise crossed pair.

Equal-amplitude transverse components in quadrature give circular polarisation. Same/opposite phase gives linear; unequal quadrature components give elliptical. A quarter-wave plate needs the appropriate input direction.

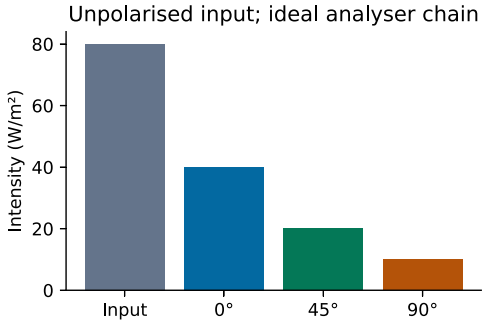
Brewster: $\tan\theta_B = n_2/n_1$ from the normal. Reflected p vanishes; reflected s survives. Use Snell's law for refraction; do not substitute the total-internal-reflection critical-angle rule.

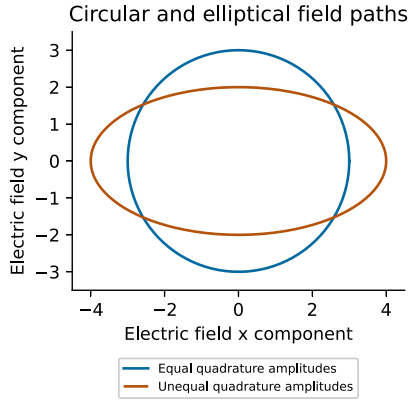
■ Worked method

An ideal first polariser transmits half of unpolarised incident intensity I0. Each later analyser uses the angle from the immediately preceding axis. For axes 0, 30 and 90 degrees,

$$I_f = \tfrac{1}{2} I_0 \cos^2(30^\circ)\cos^2(60^\circ) = 3I_0/32.$$

The field after each filter points along that filter's axis. Using only the first and last axes would predict zero incorrectly.





2 Practice

2.1 Choose one answer. Unpolarised intensity I_0 passes ideal axes 0°, 30°, 90°. Final intensity is: [1]

- **A** 0
- **B** $I_0/8$
- **C** $3I_0/32$
- **D** $3I_0/8$
- **E** $I_0/2$

2.2 Choose one answer. At one point E_x=2 cosωt and E_y=2 sinωt. The polarisation is: [1]

- **A** Linear along x
- **B** Linear at 45°
- **C** Circular
- **D** Unpolarised
- **E** Longitudinal

2.3 Polarised input intensity 24 W/m^2 meets an ideal analyser at 60°. Output is _____ W/m^2. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: At Brewster incidence for the ideal dielectric model, reflected light from unpolarised input is polarised perpendicular to the plane of incidence. [2]

3 Exam-style questions

3.1 Insert one ideal analyser at angle theta between two crossed ideal polarisers. Derive final intensity for unpolarised input I0 and find the angle giving maximum output. Compare with removing the middle analyser. [4]

Solutions

2.1 C The first output is $I_0/2$; later angles are 30° and 60°. Multiply (1/2)(3/4)(1/4)=3/32.

Compare the rejected options: 0; $I_0/8$; $3I_0/8$; $I_0/2$. Use the stated model and conditions.

2.2 C Equal nonzero amplitudes with quarter-cycle phase give constant magnitude and a rotating transverse direction.

Compare the rejected options: Linear along x; Linear at 45°; Unpolarised; Longitudinal. Use the stated model and conditions.

2.3 6. $24 \cos^2 60° = 24/4 = 6$.

2.4 True. The reflected p component is zero; the surviving s electric field is perpendicular to the incidence plane.

3.1 $I_f = (I_0/2)\cos^2\theta \sin^2\theta = (I_0/8)\sin^2(2\theta)$. The maximum is I0/8 at 45 degrees between the outer axes (modulo equivalent axes). Removing the middle analyser gives zero. It changes polarisation while reducing intensity; it does not create energy. Award chain, identity, optimum and comparison/interpretation.