

30.1 RC and RL transients, impedance and resonance

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS time constant • impedance

Objective

Solve RC charging and discharging with stated initial conditions; Solve RL current response and account for stored energy; Use complex impedance, phase and resonance in sinusoidal circuits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For RC, $R \, dq/dt + q/C = V_s$ and $\tau = RC$. $V_C = V_s + (V_C(0) - V_s) \exp(-t/\tau)$. An uncharged capacitor reaches 63.2% after τ ; discharge leaves 36.8%. Voltage is continuous without impulsive current.

For RL, $L \, dI/dt + RI = V_s$ and $\tau = L/R$. $I = V_s/R + (I(0) - V_s/R) \exp(-t/\tau)$. Current is continuous for finite voltage. Stored energy $LI^2/2$ becomes resistor heat during decay.

Using $\exp(i\omega t)$, series RLC has $Z = R + iX$ with $X = \omega L - 1/(\omega C)$. $I_{\text{rms}} = V_{\text{rms}}/\sqrt{R^2 + X^2}$; current lags for $X > 0$ and leads for $X < 0$. Add complex impedances, not magnitudes.

Series driven resonance has $\omega_0 = 1/\sqrt{LC}$, $X = 0$ and maximal current. Mean power is $I_{\text{rms}}^2 R$. Damped free oscillation instead has frequency $\sqrt{1/(LC) - (R/(2L))^2}$ when underdamped. Distinguish the drive and regime.

■ Worked method

An RC circuit switches to fixed source voltage V_s . Charge conservation and Kirchhoff's law give

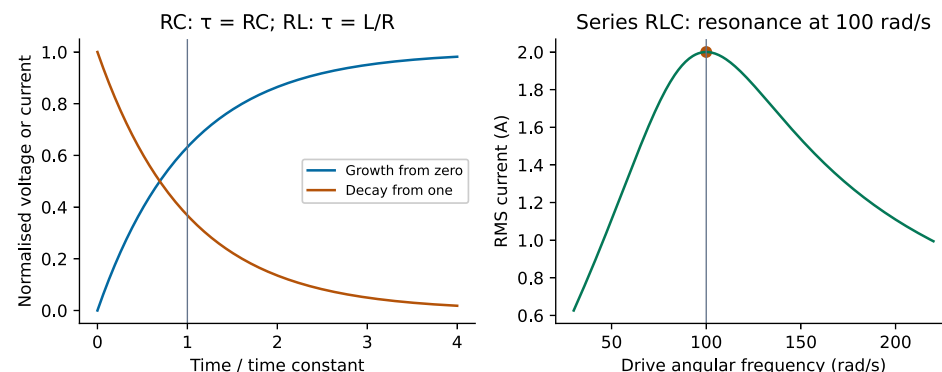
$$RC \dot{V}_C + V_C = V_s, \quad V_C(t) = V_s + [V_C(0) - V_s] e^{-t/(RC)}.$$

For $R = 2$ kilohms, $C = 1$ millifarad, $V_s = 12$ V and initial voltage zero,

$$\tau = RC = (2000 \, \Omega)(0.001 \, \text{F}) = 2.0 \, \text{s}.$$

$$V_C(\tau) = V_s(1 - e^{-1}) = (12 \, \text{V})(1 - e^{-1}) = 7.59 \, \text{V}.$$

The time constant controls the remaining difference from the final state.



2 Practice

2.1 Choose one answer. After one time constant, an initially uncharged RC capacitor driven by a constant source reaches approximately: [1]

- A 36.8% of final voltage
- B 50% of final voltage
- C 63.2% of final voltage
- D 86.5% of final voltage
- E 100% of final voltage

2.2 Choose one answer. Using $\exp(i\omega t)$, a series circuit with $X = \omega L - 1/(\omega C) > 0$ has: [1]

- A Current leading voltage
- B Current lagging voltage
- C Voltage and current necessarily in phase
- D Zero resistor dissipation
- E Impedance magnitude $R + X$

2.3 An RL circuit has $L = 0.6$ H and $R = 3 \, \Omega$. Its time constant is _____ s. Show your relation and units. [2]

[2]

[5]

Solutions

2.1 C $V_C/V_s = 1 - \exp(-1) \approx 0.632$; $\exp(-1)$ is the remaining difference.

Compare the rejected options: 36.8% of final voltage; 50% of final voltage; 86.5% of final voltage; 100% of final voltage. Use the stated model and conditions.

2.2 B Z has a positive phase; $I = V/Z$ has a negative phase relative to voltage.

Compare the rejected options: Current leading voltage; Voltage and current necessarily in phase; Zero resistor dissipation; Impedance magnitude $R + X$. Use the stated model and conditions.

2.3 0.2. $\tau = L/R = 0.6/3 = 0.2$ s.

2.4 True. Its ideal voltage/current phase is a quarter cycle; average power is zero while $LI^2/2$ varies.

3.1 With $\tau = RC$, $V_C = -4 + 12e^{-t/\tau}$ volts. Zero crossing satisfies $e^{-t/\tau} = 1/3$, so $t = \tau \ln 3$. $I(0) = (V_s - V_C(0))/R = -12\text{ V}/R$. Finite current cannot change capacitor charge instantaneously, so voltage begins at 8 V. Award solution, endpoint check, crossing, signed current and continuity.