29.1 Polarisation, magnetisation and material fields

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS electric polarisation • magnetisation

Objective

Relate free and bound charge to D, E and polarisation; Compare fixed-charge and fixed-voltage dielectric changes; Use magnetic constitutive response and free-current boundary conditions. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

P is electric dipole moment per volume. D=ε_0E+P obeys $\nabla \cdot$ D=ρ_free. Bound charges obey ρ_b=$-\nabla \cdot$ P and σ_b=P $\cdot$ n with outward n. Uniform P can give surface charge but no bound bulk charge.

For a linear isotropic dielectric, P=$\varepsilon_0\chi$_eE and ε_r=1+χ_e. In a fully filled large parallel-plate capacitor, D=σ_free and E=σ_free/ε. A dielectric need not have zero internal E.

C=εA/d. At fixed Q, V=Q/C and U=Q^2/(2C) fall as C rises. At fixed V, Q=CV and U=CV^2/2 rise. The voltage source exchanges energy. Normal D jumps by σ_free; static tangential E is continuous.

M is magnetic dipole moment per volume; B=μ_0(H+M). Linear M=χ_mH gives B=μ_0(1+χ_m)H. A long solenoid has H≈nI. Normal B is continuous; tangential H jumps with free surface current. Hysteresis needs more than constant susceptibility.

■ Worked method

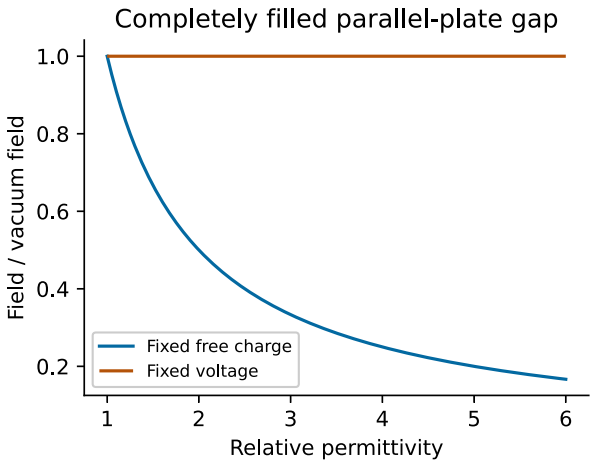
A capacitor is disconnected before inserting a linear dielectric with relative permittivity 4. Free charge stays fixed; capacitance becomes four times its original value.

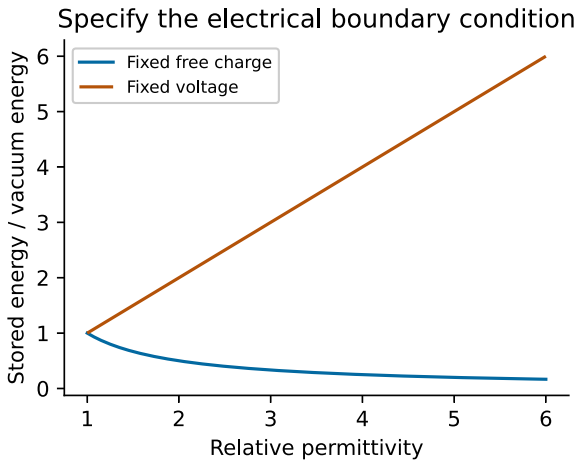
$$V' = Q/(4C) = V/4, \qquad U' = Q^2/(2 \times 4C) = U/4.$$

If an ideal battery stays connected instead, V is fixed:

$$Q' = 4CV = 4Q, \qquad U' = \tfrac{1}{2}(4C)V^2 = 4U.$$

The source exchanges energy. Do not compare isolated charge and fixed voltage as the same experiment.





2 Practice

2.1 Choose one answer. An isolated ideal parallel-plate capacitor is fully filled with a linear dielectric of ε_r=3. Which pair gives voltage and stored-energy ratios? [1]

- **A** 3 and 3
- **B** 1/3 and 1/3
- **C** 1 and 3
- **D** 1/3 and 3
- **E** 1 and 1

2.2 Choose one answer. Uniform electric polarisation in a finite dielectric necessarily implies: [1]

- **A** Nonzero bound volume charge everywhere
- **B** Zero bound surface charge everywhere
- **C** Possible bound surface charge with zero bound bulk charge
- **D** Zero internal electric field as in a metal
- **E** Identical numerical B and H

2.3 In a linear magnetic material with H=500 A/m and χ_m=0.2, M=_____ A/m. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Normal electric field must be continuous across every dielectric interface even if the permittivities differ. [2]

3 Exam-style questions

3.1 A slab fills half the thickness of a parallel-plate capacitor and has permittivity 3 epsilon0; the remaining thickness is vacuum. Find C relative to the empty capacitor, and the two normal fields at fixed free charge Q. Ignore edge effects. [5]

Solutions

2.1 B Free Q stays fixed and C triples; both V=Q/C and U=Q^2/(2C) become one third.

Compare the rejected options: 3 and 3; 1 and 3; 1/3 and 3; 1 and 1. Use the stated model and conditions.

2.2 C ρ_b=$-\nabla \cdot$ P=0 for uniform P, but σ_b=P $\cdot$ n can be nonzero.

Compare the rejected options: Nonzero bound volume charge everywhere; Zero bound surface charge everywhere; Zero internal electric field as in a metal; Identical numerical B and H. Use the stated model and conditions.

2.3 100. M=χ_mH=0.2 $\cdot$ 500=100.

2.4 False. With no free surface charge, normal D is continuous; E=D/ε can differ.

3.1 Layers are in series. With no free interface charge, $D = Q/A$ is common. $E_{vac} = Q/(\epsilon_0 A)$ and $E_d = Q/(3\epsilon_0 A)$. $V = (d/2)(E_{vac} + E_d) = 2Qd/(3\epsilon_0 A)$, hence $C = 3\epsilon_0 A/(2d) = 3C_0/2$. Award common D, each field, voltage sum and capacitance. Normal E is discontinuous; D is continuous.