# 24.1 Scattering, degeneracy and Pauli operators

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

**KEY WORDS** reflection coefficient • degeneracy

## Objective

Match travelling waves and fluxes across finite potential changes; Count degenerate isotropic-oscillator states; Evaluate Pauli products and distinguish anticommutation from equality. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

For constant potential with E>V, k=sqrt[2m(E−V)]/$\hbar$ and solutions travel as exp(±ikx). At finite steps of equal mass, $\psi$ and $\psi'$ are continuous. No incoming wave from the right leaves only a right-travelling transmitted wave. A finite well returning to its external potential has equal incident/transmitted external k.

Step matching gives r=(k1−k2)/(k1+k2), t=2k1/(k1+k2). Reflection R=|r|$^2$; transmission T=(k2/k1)|t|$^2$ uses probability current. R+T=1 for a lossless step. Downward steps can reflect; raw squared transmitted amplitude is not always its current fraction.

A spin-zero 3D isotropic oscillator has energy (N+3/2)$\hbar\omega$, N=n_x+n_y+n_z≥0. Ordered nonnegative triples number (N+1)(N+2)/2. For N=2, permutations of (2,0,0) and (1,1,0) give six states. Extra spin/identical-particle constraints require separate counting.

Pauli matrices $\sigma$_x=[[0,1],[1,0]], $\sigma$_y=[[0,−i],[i,0]], $\sigma$_z=[[1,0],[0,−1]] each square to I. Cyclic products give $\sigma$_x$\sigma$_y=i$\sigma$_z; reversed order changes sign. Thus distinct matrices anticommute, and $\sigma$_x$\sigma$_z=−i$\sigma$_y. Physical spin is S_i=$\hbar\sigma$_i/2.

### ■ Worked method

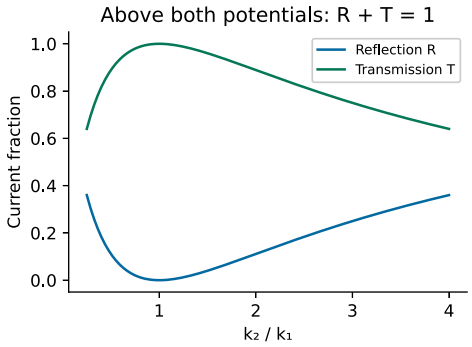
At a finite potential step, match wavefunction and derivative for the same particle mass.

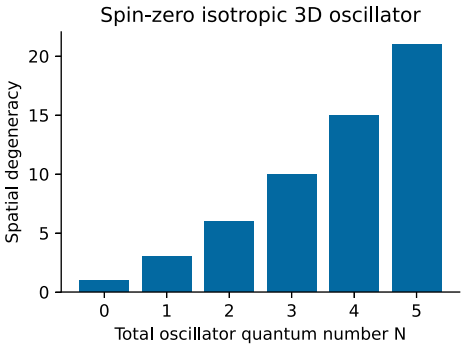
$$r = \frac{k_1 - k_2}{k_1 + k_2}, \qquad t = \frac{2k_1}{k_1 + k_2}.$$

Transmission probability uses current, not amplitude alone. For $k_2 = 3k_1$, $r = -1/2$, $t = 1/2$.

$$R = |r|^2 = 1/4, \qquad T = (k_2/k_1)|t|^2 = 3/4, \qquad R + T = 1.$$

Reflection occurs even for a downward step. Its phase sign disappears when taking probability.




## 2 Practice

**2.1** Choose one answer. For a lossless potential step with k2=2k1, reflection probability is: [1]

- **A** 0
- **B** 1/9
- **C** 1/3
- **D** 4/9
- **E** 1

**2.2** Choose one answer. A spin-zero three-dimensional isotropic oscillator at E=(5/2)$\hbar\omega$ has degeneracy: [1]

- **A** 1
- **B** 2
- **C** 3
- **D** 4
- **E** 6

**2.3** For a spin-zero isotropic oscillator with N=4, spatial degeneracy is _____. Show your relation and units. [2]

**2.4** Decide whether this claim is true. Justify it: Distinct Pauli matrices commute with each other. [2]

## 3 Exam-style questions

**3.1** A spin-half particle has state $(\sqrt{3}, i)^T/2$ in the z-spin basis. Check normalisation and find probabilities for Sz and Sx outcomes. [5]

## Solutions

**2.1 B** r=(1−2)/(1+2)=−1/3; R=1/9.

Compare the rejected options: 0; 1/3; 4/9; 1. Use the stated model and conditions.

**2.2 C** N=1 gives triples (1,0,0),(0,1,0),(0,0,1).

Compare the rejected options: 1; 2; 4; 6. Use the stated model and conditions.

**2.3** 15. (N+1)(N+2)/2=5 · 6/2=15.

**2.4** False. Their products change sign on reversing order; they anticommute.

**3.1** Norm is $(3+1)/4 = 1$. Sz gives $+\hbar/2$ with probability 3/4 and $-\hbar/2$ with probability 1/4. The x eigenstates are $(1, \pm 1)^T/\sqrt{2}$. Their amplitudes are $(\sqrt{3} \pm i)/(2\sqrt{2})$, each with squared modulus 1/2. Award norm, z results/probabilities, x eigenbasis, projections and x probabilities. Complex conjugation is required in the inner product; the imaginary relative phase matters.