

16.1 Acoustic Doppler echoes and standing-wave boundaries

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS displacement antinode • Doppler factor

Objective

Apply source and observer Doppler factors in the medium frame; Derive allowed pipe frequencies from displacement boundary conditions; Distinguish frequency, wavelength and boundary changes. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For sound in a stationary medium, approaching source speed u_s gives $f_{\text{received}} = f c / (c - u_s)$. Approaching observer speed u_o gives $f_{\text{received}} = f(c + u_o) / c$. Source motion changes wavefront spacing; observer motion changes encounter rate. These roles are not interchangeable; use medium-frame velocities and reverse signs for recession.

A stationary wall receives $f c / (c - u)$, then reflects that frequency unchanged in the medium frame. The approaching driver receives another factor $(c + u) / c$, giving $f_{\text{echo}} = f(c + u) / (c - u)$. At small u/c the fractional shift is about $2u/c$; a moving reflector needs its own analysis.

Ideal open ends have displacement antinodes/pressure nodes; closed ends have displacement nodes/pressure antinodes. Both-open modes have $f_n = nc / (2L)$. One-closed modes have $f_n = (2n - 1)c / (4L)$. These formulas assume ideal boundaries; real end corrections require information not supplied by the simple model.

Closing one end halves the fundamental and permits only odd multiples of that new value. Old both-open lines are even multiples of the new fundamental, so none remains in the ideal closed spectrum. Medium wave speed stays unchanged while frequency and wavelength adapt to the new boundaries.

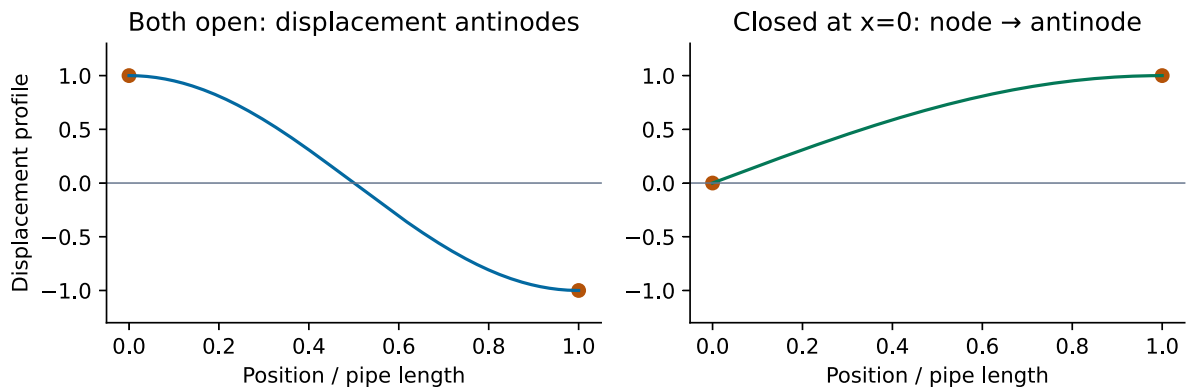
■ Worked method

A siren approaches a stationary wall in still air. Treat outward and return journeys separately. The wall first receives $f_w = f c / (c - u)$; reflection keeps this frequency in the medium frame. The moving listener then receives $f_e = f_w(c + u) / c$.

$$f_e = f \frac{c + u}{c - u} = (600 \text{ Hz}) \frac{340 \text{ m/s} + 20 \text{ m/s}}{340 \text{ m/s} - 20 \text{ m/s}} = 675 \text{ Hz}.$$

This Doppler effect uses velocities relative to the medium. It is not the light-wave formula.

Ideal fundamental modes; $f_{\text{closed}} = f_{\text{open}} / 2$



2 Practice

2.1 Choose one answer. A moving siren approaches a stationary wall at speed u in still air with speed c . The driver's echo frequency divided by emitted frequency is: [1]

- **A** $c/(c-u)$
- **B** $(c+u)/c$
- **C** $(c+u)/(c-u)$
- **D** $(c-u)/(c+u)$
- **E** 1

2.2 Choose one answer. An ideal both-open pipe has fundamental 120 Hz. After closing one end, its first three allowed frequencies are: [1]

- **A** 120,240,360 Hz
- **B** 120,360,600 Hz
- **C** 60,120,180 Hz
- **D** 60,180,300 Hz
- **E** 240,480,720 Hz

2.3 For $c=330$ m/s and $u=30$ m/s toward a stationary wall, $f_{\text{echo}}/f=$ _____ (decimal). Show your relation and units. [2]

Solutions

2.1 C The outbound moving-source factor and return moving-observer factor multiply.

Compare the rejected options: $c/(c-u)$; $(c+u)/c$; $(c-u)/(c+u)$; 1. Use the stated model and conditions.

2.2 D The new fundamental is 60 Hz and allowed multiples are odd.

Compare the rejected options: 120,240,360 Hz; 120,360,600 Hz; 60,120,180 Hz; 240,480,720 Hz. Use the stated model and conditions.

2.3 1.2. $(330+30)/(330-30)=360/300=1.2$.

2.4 False. The one-closed spectrum has no even multiples of its new fundamental; that original line is absent.

3.1 The reflector receives $f(c+u)/c$ as a moving observer and re-emits toward the receiver as a moving source. Therefore $f_r = f(c+u)/(c-u)$. $r(c-u) = c+u$ gives $u = c(r-1)/(r+1)$. For $u \ll c$, $r \approx 1 + 2u/c$. Award two factors, product, inversion and limit. The equality with the moving-siren echo ratio does not make source and observer roles interchangeable.