15.1 Maxwell waves, energy flow and boundary conditions

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS displacement current • Poynting vector

Objective

Use displacement current and material wave-speed relations; Determine wave-field directions and radiated energy flow; Apply Maxwell boundary conditions without confusing field components. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Vacuum Ampere–Maxwell law is $\oint B \cdot dl = \mu 0(I + \varepsilon 0 d\Phi E/dt)$. Displacement current depends on changing electric flux, not magnetic flux. It makes capacitor-loop results consistent for surfaces cutting the wire or crossing the gap. For a uniform gap field its value is $\varepsilon 0 A\, dE/dt$.

In a homogeneous linear lossless medium, $v = 1/\text{sqrt}(\mu\varepsilon)$, $n = \text{sqrt}(\mu r \varepsilon r)$. For nonmagnetic material, $v = c/\text{sqrt}(\varepsilon r)$. Use constants appropriate to the frequency if dispersion matters. $E/B = v$ in this simple medium, and c in vacuum.

For propagation unit vector n, $B = (n \times E)/v$. Phase $kz - \omega t$ travels +z; $kz + \omega t$ travels −z. A +z wave with E along x+y has B along −x+y. Poynting vector $E \times B/\mu$ gives energy-flow direction; far-zone radiation from an accelerating charge carries energy outward.

Because $\nabla \cdot B = 0$, normal B is continuous across a boundary. Zero interior B in an ideal Meissner model therefore gives zero exterior normal B; a tangential field can remain with a surface current. This conclusion uses the specified model, not an assumption of zero field everywhere. Normal D can instead jump by free surface charge.

■ Worked method

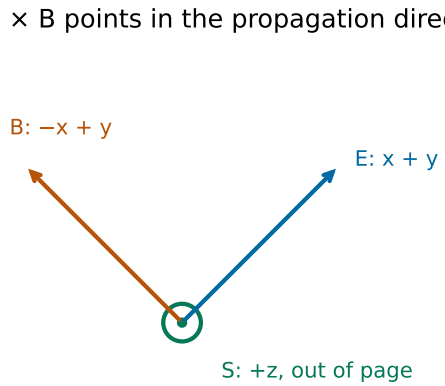
A plane wave travels along positive z in a linear lossless nonmagnetic dielectric . With relative permittivity 9,

$$v = c/\sqrt{\epsilon_r} = (3.00 \times 10^8\,\text{m/s})/3 = 1.00 \times 10^8\,\text{m/s}.$$

If E points along positive x, $\mathbf{B} = \hat{\mathbf{z}} \times \mathbf{E}/v$ points along positive y. For $E0 = 200$ V/m,

$$B_0 = E_0/v = (200\,\text{V/m})/(1.00 \times 10^8\,\text{m/s}) = 2.00\,\mu\text{T}.$$

E × **H** points along propagation. Check that material constants apply at the wave frequency.



2 Practice

2.1 Choose one answer. For a +z plane wave whose electric field is along +x, magnetic field points: [1]

- **A** +x
- **B** −x
- **C** +y
- **D** −y
- **E** +z

2.2 Choose one answer. If interior B is zero at an ideal Meissner boundary, a nonzero outside B must be: [1]

- **A** Normal to the surface
- **B** Tangent to the surface
- **C** Zero at all outside points
- **D** Time dependent
- **E** Parallel to E in every case

2.3 In a nonmagnetic lossless dielectric with εr=16, wave speed is _____ times c (decimal). Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: For capacitor charging, displacement current depends on the rate of change of electric flux rather than magnetic flux. [2]

3 Exam-style questions

3.1 A vacuum plane wave travels along negative x, with electric field along positive z. Determine magnetic-field direction and average energy flux for peak electric amplitude E0. A perfect absorber of area A is normal to the beam. Find its mean force. [4]

Solutions

2.1 C z×x=y, ensuring E×B points +z.

Compare the rejected options: +x; −x; −y; +z. Use the stated model and conditions.

2.2 B Normal B is continuous and therefore zero; a tangential component can remain.

Compare the rejected options: Normal to the surface; Zero at all outside points; Time dependent; Parallel to E in every case. Use the stated model and conditions.

2.3 0.25. v/c=1/sqrt(16)=1/4.

2.4 True. The gap term is ε0dΦE/dt in vacuum.

3.1 $\mathbf{B} = \hat{\mathbf{n}} \times \mathbf{E}/c$ points along positive y since $(-\hat{x}) \times \hat{z} = \hat{y}$. $\langle S \rangle = E_0^2/(2\mu_0 c) = \epsilon_0 c E_0^2/2$ for a sinusoid. Absorbed power is $\langle S \rangle A$ and force magnitude is $\langle S \rangle A/c$, along negative x. Award magnetic direction, mean flux, power and force/direction. A perfect reflector would double the force; it is a different boundary condition.